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ROLE OF PARTIAL FUNCTIONALS IN THE STUDY OF VARIATIONAL SYSTEMS

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ABSTRACT. Applying techniques originally developed for systems lacking a variational structure, we establish conditions for the existence of solutions in systems that possess this property but their energy functional is unbounded both above and below. We show that, in general, our conditions differ from those in the classical mountain pass approach by Ambrosetti–Rabinowitz when dealing with systems of this type. Our theory is put into practice in the context of a coupled system of Stokes equations with reaction terms, where we establish sufficient conditions for the existence of a solution. The systems under study are intermediary between gradient-type systems and Hamiltonian systems.

1. Introduction and preliminaries

Many real-world processes can be represented by equations or systems of equations. However, solving these problems can be quite challenging. Over time, various techniques have been developed, with the critical point technique being one of the most significant. This technique is important because it simplifies

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