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ON CONLEY THEORY FOR GENERALIZED GUTIERREZ–SOTOMAYOR FLOWS

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ABSTRACT. This paper presents a study, based on Conley’s theory, of continuous flows on singular surfaces that admit singularities such as n -sheet cones, n -sheet cross-caps, n -sheet double crossings, n -sheet triple crossings, and mixed Gutierrez–Sotomayor (GS) singularities. These flows are referred to as generalized Gutierrez–Sotomayor (GS) flows. The Conley index for each type of singularity is computed. Furthermore, the results from previous works on the local and global existence of Lyapunov functions are extended to encompass generalized GS singularities. Necessary and sufficient conditions for defining a generalized GS flow on an isolating block are established. Additionally, an alternative formula, expressed in terms of a deeper dynamical perspective, for computing the Euler–Poincaré characteristic of generalized GS manifolds is introduced.

1. Introduction

Conley Index Theory, named after Charles Conley, focuses on investigating isolated invariant sets of continuous flows defined on a topological space. The primary idea is to characterize the topological structure of these sets by analyzing the dynamics in their surroundings, [3], [4]. Conley’s theory can be

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