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STRONG NIELSEN EQUIVALENCE ON THE PUNCTURED DISC

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Abstract. Let f be an orientation-preserving homeomorphism of the 2-disc $\mathbb{D}^2$ that fixes the boundary pointwise and leaves invariant a finite subset in the interior of $\mathbb{D}^2$. We study the strong Nielsen equivalence of periodic points of such homeomorphisms f and we give a necessary and sufficient condition for two periodic points to be strong Nielsen equivalent in the context of braid theory. In addition, we present an application of our result to the trace formula given by Jiang–Zheng, deducing that the obtained forced periodic orbits belong to different strong Nielsen classes.

1. Introduction

A classical problem related to Nielsen theory deals with the question of determining the minimum number of fixed points among all maps homotopic to a given map f from a compact space to itself. In particular, let X be a compact connected polyhedron, $f\colon X \to X$ a continuous self-map, and let $\mathrm{Fix}(f) = \{x \in X \mid f(x) = x\}$ be the set of fixed points of f. One is interested in studying the so-called minimal number of the map f denoted by $\mathrm{MF}[f]$ and defined as $\mathrm{MF}[f] = \min\{\#\mathrm{Fix}(g) \mid g \sim f\}$, where $\sim$ means homotopic. We define an equivalence relation on the set of fixed points of f in the following way. Two fixed points belong in the same fixed point class if there exists a path γ joining them, such that $f(\gamma) \sim \gamma$ keeping the endpoints fixed during the homotopy.

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