

## MULTIPLICITY OF SOLUTIONS TO CRITICAL $p$ -LAPLACE EQUATIONS INVOLVING A HARDY POTENTIAL

DIEM HANG T. LE — PHUONG LE

ABSTRACT. In this paper, we prove the existence of at least  $N$  pairs of nontrivial solutions to the doubly critical quasilinear elliptic problem

$$-\Delta_p u - \frac{\lambda}{|x|^p} |u|^{p-2} u = a(x) |u|^{p-2} u + |u|^{p^*-2} u$$

in  $\mathbb{R}^N$ , as well as in smooth bounded domains, where  $1 < p < N$ ,  $0 < \lambda < ((N-p)/p)^p$  and  $a$  is strictly positive in a small ball. Our results hold under the assumption that  $N \geq p^2$  and  $\lambda$  and  $\|a^+\|_{L^{N/p}}$  are small enough. To circumvent difficulties due to the lack of compactness of the problem, we combine Krasnosel'skii's genus with a recent classification result by Oliva, Sciunzi, Vaira (J. Math. Pures Appl. **140** (2020), 89–109) and global compactness results by Li, Guo, Niu (Nonlinear Anal. **74** (2011), no. 4, 1445–1464).

### 1. Introduction

Let  $1 < p < N$  and  $0 < \lambda < ((N-p)/p)^p$ . We study the existence of nontrivial solutions to the doubly critical  $p$ -Laplace problem

$$(1.1) \quad -\Delta_p u - \frac{\lambda}{|x|^p} |u|^{p-2} u = a(x) |u|^{p-2} u + |u|^{p^*-2} u, \quad u \in \mathcal{D}^{1,p}(\mathbb{R}^N),$$

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