

BRÉZIS–KATO TYPE REGULARITY RESULTS FOR HIGHER ORDER ELLIPTIC OPERATORS

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ABSTRACT. We prove Brézis–Kato regularity type results for solutions of the higher order nonlinear elliptic equation

$$Lu = g(x, u) \quad \text{in } \Omega$$

with an elliptic operator L of $2m$ order with variable coefficients and a Carathéodory function $g: \Omega \times \mathbb{C} \rightarrow \mathbb{C}$, where $\Omega \subset \mathbb{R}^N$ is an open set with $N > 2m$.

1. Notations and intorduction

The aim of this paper is to generalize the Brézis–Kato theorem [3] for higher order elliptic differential operators with variable coefficients. It is intended to make the least assumptions possible concerning the coefficients of the operator so that it can serve for future references.

We consider a linear differential operator $L = (-1)^m \sum_{|\alpha| \leq 2m} a_\alpha(x) D^\alpha$ with variable complex-valued coefficients defined in an arbitrary open subset Ω of \mathbb{R}^N . Here for every multi-index $\alpha = (\alpha_1, \dots, \alpha_N)$ we set

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \dots \partial x_N^{\alpha_N}} \quad \text{and} \quad \xi^\alpha = \xi_1^{\alpha_1} \dots \xi_N^{\alpha_N},$$

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