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WELL-ORDERED AND NON-WELL-ORDERED LOWER AND UPPER SOLUTIONS FOR PERIODIC $2N$-DIMENSIONAL SYSTEMS

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Abstract. In this paper we consider a class of periodic problems associated with $2N$-dimensional systems of differential equations. Our aim is to generalize the theory of lower and upper solutions following the way paved in previous works. After a careful analysis of the dynamics in the phase space, the proofs take advantage of topological degree arguments.

1. Introduction

Considering the scalar second order equation with periodic boundary conditions

$$\begin{cases} x'' = g(t,x,x'), \\ x(0) = x(T), \quad x'(0) = x'(T), \end{cases} \tag{1.1}$$

where $g\colon \mathbb{R}^3 \to \mathbb{R}$ is continuous, two functions $\alpha,\beta\colon \mathbb{R} \to \mathbb{R}$ are said to be respectively a lower and an upper solution for (1.1) if they are of class $\mathcal{C}^2$, T-periodic, and

$$\alpha''(t) \ge g(t,\alpha(t),\alpha'(t)), \quad \beta''(t) \le g(t,\beta(t),\beta'(t)), \quad \text{for every } t \in \mathbb{R}.$$

If the pair is well-ordered, namely $\alpha(t) \le \beta(t)$ for every $t \in \mathbb{R}$, and we assume some Nagumo-type conditions [17], then there exists a solution x of (1.1) such

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