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POSITIVE SOLUTION OF QUASILINEAR ELLIPTIC EQUATIONS IN $\mathbb{R}^N$ WITH A BOUNDED QUASILINEARITY

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Abstract. Consider the quasilinear elliptic equation

$$-\mathrm{div}(\mathcal{A}(u)\nabla u) + \frac{1}{2}\mathcal{A}'(u)|\nabla u|^2 + V(x)u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N,$$

where $\mathcal{A} \in C^1(\mathbb{R},\mathbb{R})$ is a positive bounded function, V is a given potential and I_α denotes the Riesz potential with $0 < \alpha < N$. While most existing works in the literature are concerned with the case where $\mathcal{A}$ is unbounded, little is known about the case where $\mathcal{A}$ is bounded. Under some general conditions on $\mathcal{A}$ and V, we establish the existence of a positive solution for the above equation by variational approach.

1. Introduction

Nonlinear Schrödinger equation of the form

$$i\psi_t + \mathrm{div}(\mathcal{A}(|\psi|)\nabla\psi) - \frac{1}{2}\nabla[\mathcal{A}(|\psi|)]\cdot\nabla\psi - W(x)\psi + f(x,|\psi|)\psi = 0 \tag{1.1}$$

appears in many fields of physics, where $i = \sqrt{-1}$ is the imaginary unit, ψ is a complex-valued wave function, W is a given potential, and $\mathcal{A}$, f are real-valued functions. In the special case where $\mathcal{A}$ is a positive constant, (1.1) is nothing but the classical semilinear Schrödinger equation, which has been well studied

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