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## FIXED POINT THEORY ON BANACH SPHERES

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**ABSTRACT.** In this paper, we consider and study the concept of the Banach sphere. The usual spherical distance of the unit sphere of a Hilbert space is defined by its inner product and the inverse of the cosine function. Therefore, we cannot apply this notion to Banach spheres in general. We first introduce a two-variable function like a spherical distance and a notion of convex combination on a Banach sphere. After that, we define a projection onto a subset of a Banach sphere, and prove a fixed point theorem and fixed point approximation for a mapping having a kind of nonspreadingness. Our work is a challenge to construct optimisation theory on spherical planes without geodesics.

### 1. Introduction

The problem of finding a fixed point of nonlinear mappings on Hilbert or Banach spaces is one of the most important problems in convex analysis, and it has been investigated by many researchers. Recently, fixed point theory is discussed on nonlinear spaces, for instance, infinite dimensional spheres and hyperboloids. Such planes are generalised to a geodesic metric space with curvature bounded above by a real number  $\kappa$  which is called a  $CAT(\kappa)$  space. In  $CAT(\kappa)$  spaces, there are many works about fixed point existence and approximation. For instance, the following is a pioneer result of fixed point theory on geodesic spaces:

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