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EXISTENCE OF EIGENVALUES FOR ANISOTROPIC AND FRACTIONAL ANISOTROPIC PROBLEMS VIA LUSTERNIK–SCHNIRELMANN THEORY

Ignacio Ceresa Dussel — Julián Fernández Bonder

Abstract. In this work, our interest lies in proving the existence of critical values of the following Rayleigh-type quotients

$$\mathcal{Q}_{\mathbf{p}}(u) = \frac{\|\nabla u\|_{\mathbf{p}}}{\|u\|_{\mathbf{p}}} \quad \text{and} \quad \mathcal{Q}_{\mathbf{s},\mathbf{p}}(u) = \frac{[u]_{\mathbf{s},\mathbf{p}}}{\|u\|_{\mathbf{p}}},$$

where $\mathbf{p} = (p_1, \ldots, p_n)$, $\mathbf{s} = (s_1, \ldots, s_n)$ and

$$\|\nabla u\|_{\mathbf{p}} = \sum_{i=1}^{n} \|u_{x_i}\|_{p_i}$$

is an anisotropic Sobolev norm, $[u]_{\mathbf{s},\mathbf{p}}$ is a fractional version of the same anisotropic norm, and

$$\|u\|_{\mathbf{p}} = \left(\int_{\mathbb{R}} \left(\ldots \left(\int_{\mathbb{R}} |u|^{p_1}\, dx_1 \right)^{p_2/p_1} dx_2 \ldots \right)^{p_n/p_{n-1}} dx_n \right)^{1/p_n}$$

is an anisotropic Lebesgue norm. Using the Lusternik–Schnirelmann theory, we prove the existence of a sequence of critical values and we also find an associated Euler–Lagrange equation for critical points. Additionally, we analyze the connection between the fractional critical values and its local counterparts.

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