
Topological Methods in Nonlinear Analysis
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$p(x)$ -BIHARMONIC EQUATIONS INVOLVING $(h, r(x))$ -HARDY SINGULAR COEFFICIENTS WITH NO-FLUX BOUNDARY CONDITIONS

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ABSTRACT. In this article, we investigate $p(x)$ -biharmonic equations involving two kinds of different Hardy potentials, in which the $r(x)$ -Hardy potentials are seldom mentioned in previous papers. New criteria for the existence of generalized solutions are reestablished when the nonlinear terms satisfying appropriate assumptions. The results are based on variational methods and the theory of variable exponent Sobolev spaces.

1. Introduction

Elliptic partial differential equations with variable exponents allow for fine qualitative analyses of real-world phenomena in applied sciences, such as heat transfer in conducting materials, non-Newtonian fluids, and electrorheological fluids. Electrorheological fluids were first studied in the last century when spontaneously stopping fluids were discovered. The first major discovery related to electrorheological fluids was made by Winslow in the literature [19] in 1949. The remarkable properties of such fluids include change of viscosity in the presence of an electric field. We would also like to mention [11], where the authors

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