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HOPF BIFURCATION AND STABILITY ANALYSIS FOR A DELAYED EQUATION WITH φ-LAPLACIAN

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Abstract. A formal framework for the analysis of Hopf bifurcations for a kind of delayed equation with φ-Laplacian and with a discrete time delay is presented, thus generalizing known results for the sunflower equation given by Somolinos in 1978. Also, under appropriate assumptions we prove the gradient-like behavior of the equation which, in turn, implies the non-existence of nonconstant periodic solutions. Our conditions improve previous results known in the literature for the standard case $\varphi(x) = x$.

1. Introduction

The purpose of this article is to study the following equation

$$(\varphi(x'(t)))' + \frac{a}{r}\, x'(t) + \frac{b}{r}\, g(x(t-r)) = 0, \quad t \geq 0, \tag{1.1}$$

where $\varphi\colon \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism such that $\varphi(0) = 0$. Further, $a, b > 0$ are nonzero real constants a, b, and the delay r is a positive constant. Moreover, $g\colon \mathbb{R} \to \mathbb{R}$ is continuously differentiable, such that $g(0) = 0$ and $g'(0) > 0$.

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