

EXISTENCE OF POSITIVE SOLUTIONS FOR A GENERALIZED FRACTIONAL BRÉZIS–NIRENBERG PROBLEM

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ABSTRACT. In this article, we study the fractional Brézis–Nirenberg type problem on whole domain \mathbb{R}^N associated with the fractional p -Laplace operator. To be precise, we want to study the following problem:

$$(P) \quad (-\Delta_p)^s u - \lambda w |u|^{p-2} u = |u|^{p_s^*-2} u \quad \text{in } \mathcal{D}^{s,p}(\mathbb{R}^N),$$

where $s \in (0, 1)$, $p \in (1, N/s)$, $p_s^* = Np/(N - sp)$ and the operator $(-\Delta_p)^s$ is the fractional p -Laplace operator. The space $\mathcal{D}^{s,p}(\mathbb{R}^N)$ is the completion of $C_c^\infty(\mathbb{R}^N)$ with respect to the Gagliardo semi-norm. In this article, we prove the existence of a positive solution to problem (P) by allowing the Hardy weight function w to change its sign.

1. Introduction

In this paper, we study the Brézis–Nirenberg type problem associated with the fractional p -Laplace operator on the whole domain \mathbb{R}^N given by

$$(1.1) \quad (-\Delta_p)^s u - \lambda w |u|^{p-2} u = |u|^{p_s^*-2} u \quad \text{in } \mathcal{D}^{s,p}(\mathbb{R}^N),$$

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Key words and phrases. Fractional Brézis–Nirenberg problem; critical Sobolev exponent; concentration compactness; principle of symmetric criticality; positive solutions.

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