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## A CHARACTERIZATION OF $g_2$ -MINIMAL NORMAL 3-PSEUDOMANIFOLDS WITH AT MOST FOUR SINGULARITIES

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ABSTRACT. Let  $\Delta$  be a  $g_2$ -minimal normal 3-pseudomanifold. A vertex in  $\Delta$  whose link is not a sphere is called a singular vertex. When  $\Delta$  contains at most two singular vertices, its combinatorial characterization is known [9]. In this article, we present a combinatorial characterization of such a  $\Delta$  when it has three singular vertices, including one  $\mathbb{R}P^2$ -singularity, or four singular vertices, including two  $\mathbb{R}P^2$ -singularities. In both cases, we prove that  $\Delta$  is obtained from a one-vertex suspension of a surface, and some boundary complexes of 4-simplices by applying the combinatorial operations of types connected sums, vertex foldings, and edge foldings.

### 1. Introduction

The study of pseudomanifolds, and in particular normal pseudomanifolds, has been one of the central topics in combinatorial topology. Structures for such objects from a combinatorial and topological viewpoint in terms of simplicial and cell complexes are being developed by various researchers throughout the world. A basic and fundamental enumerative invariant of any  $n$ -dimensional simplicial

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