

STRONG SURJECTIONS FROM TWO-COMPLEXES WITH TRIVIAL TOP-COHOMOLOGY ONTO NONORIENTABLE SURFACES

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ABSTRACT. For every nonorientable closed surface U , we present a strong surjection $f: X \rightarrow U$, where X is a finite two-dimensional CW-complex with trivial second integer cohomology group. This provides an answer, for all nonorientable closed surfaces, to a problem in topological root theory for which we have hitherto known solutions only for the sphere, the torus, the projective plane and the Klein bottle.

1. Introduction

Since the mid-2000s, the following problem, related to topological root theory, has been studied: given a (connected) closed n -manifold M , is there a strong surjection $f: K \rightarrow M$, where K is a finite and connected n -dimensional CW-complex with $H^n(K; \mathbb{Z}) = 0$?

By a *strong surjection* (or a *strongly surjective* map) we mean a (continuous) map whose (free) homotopy class contains only surjective maps.

This problem was proposed by Daciberg Lima Gonçalves to his PhD student Claudemir Aniz, who answered the question for some 3-manifolds: the answer is negative for the tori $S^1 \times S^2$ and $S^1 \times S^1 \times S^1$, the Lens spaces $L(n, p)$ and the spherical manifolds S^3/Q_{4k} for $k = 2, 4, 8$, where $Q_{4k} = \langle x, y \mid x^k = y^2, yxy = y \rangle \subset S^3$ is the quaternion group of order $4k$; the answer is affirmative

2020 *Mathematics Subject Classification.* 55M20.

Key words and phrases. Strong surjections; two-complexes; nonorientable surfaces.

I thank FAPEMIG – Grant RED-00133-21: *Rede Mineira de Matemática*.