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MULTIPLICITY OF 2-NODAL SOLUTIONS OF THE YAMABE EQUATION

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Abstract. Given a closed Riemannian manifold (M, g), we use the gradient flow method and Sign-Changing Critical Point Theory to prove multiplicity results for 2-nodal solutions of a subcritical non-linear equation on (M, g), see (1.1) below. If (N, h) is a closed Riemannian manifold of constant positive scalar curvature our result gives multiplicity results for the Yamabe-type equation on the Riemannian product $(M \times N, g + \varepsilon h)$, for $\varepsilon > 0$ small.

1. Introduction

On a compact Riemannian manifold (M^n, g) without boundary of dimension $n \geq 3$, we consider the following equation

$$-\varepsilon^2 \Delta_g u + \left(\frac{s_g}{a_{m+n}}\,\varepsilon^2 + 1\right) u = |u|^{p_{m+n}-2} u, \tag{1.1}$$

where s_g is the scalar curvature of g, Δ_g is the Laplace Beltrami operator associated to g, $a_{m+n} = 4(n+m-1)/(n+m-2)$, $p_{n+m} = 2(n+m)/(n+m-2)$, with $m \in \mathbb{N}$. Moreover, we consider $\varepsilon > 0$ small enough so that , for some $c_\varepsilon > 0$,

$$1 + \frac{s_g}{a_{m+n}}\,\varepsilon^2 > c_\varepsilon \quad \text{in } M. \tag{1.2}$$

The study of this equation is motivated, on one hand, by the Yamabe problem on products of Riemannian manifolds. If $u\colon M \to \mathbb{R}$ is a positive solution of equation (1.1) then u solves the Yamabe equation in the product $\big(M^n \times N^m,$

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