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## THE LIMITING BEHAVIOR OF SOLUTIONS TO $p$ -LAPLACIAN PROBLEMS WITH CONVECTION AND EXPONENTIAL TERMS

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ABSTRACT. We consider, for  $a, l \geq 1$ ,  $b, s, \alpha > 0$ , and  $p > q \geq 1$ , the homogeneous Dirichlet problem for the equation  $-\Delta_p u = \lambda u^{q-1} + \beta u^{a-1} |\nabla u|^b + m u^{l-1} e^{\alpha u^s}$  in a smooth bounded domain  $\Omega \subset \mathbb{R}^N$ . We prove that under certain setting of the parameters  $\lambda$ ,  $\beta$  and  $m$  the problem admits at least one positive solution. Using this result we prove that if  $\lambda, \beta > 0$  are arbitrarily fixed and  $m$  is sufficiently small, then the problem has a positive solution  $u_p$ , for all  $p$  sufficiently large. In addition, we show that  $u_p$  converges uniformly to the distance function to the boundary of  $\Omega$ , as  $p \rightarrow \infty$ . This convergence result is new for nonlinearities involving a convection term.

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