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CHARACTERIZATION OF THE ALGEBRAIC DIFFERENCE OF SPECIAL AFFINE CANTOR SETS

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Abstract. We investigate some self-similar Cantor sets $C(l, r, p)$, which we call S-Cantor sets, generated by numbers $l, r, p \in \mathbb{N}$, $l + r < p$. We give a full characterization of the set $C(l_1, r_1, p) - C(l_2, r_2, p)$ which can take one of the form: the interval $[-1, 1]$, a Cantor set, an L-Cantorval, an R-Cantorval or an M-Cantorval. As corollaries we give examples of Cantor sets and Cantorvals, which can be easily described using some positional numeral systems.

1. Introduction

We denote by $A \pm B$ the set $\{a \pm b : a \in A,\, b \in B\}$, where $A, B \subset \mathbb{R}$. The set $A - B$ is called the algebraic difference of sets A and B. The set $A - A$ is called the difference set of a set A. We will also write $a + A$ rather than $\{a\} + A$ for $a \in \mathbb{R}$. Let $I \subset \mathbb{R}$ be an interval. We denote by $l(I)$, $r(I)$ the left and the right endpoint of I, respectively.

We say that a set $C \subset \mathbb{R}$ is a Cantor set if it is nonempty, compact, perfect and nowhere dense. For a set $C \subset \mathbb{R}$, every component of the set $\mathbb{R} \setminus C$ is called a gap of C. A component of C is called proper if it is not a singleton.

Let us recall the definitions of three types of Cantorvals (compare [12]). A perfect set $E \subset \mathbb{R}$ is called an M-Cantorval if it has infinitely many gaps and

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