

## WEAKLY NONLINEAR HYPERBOLIC DIFFERENTIAL EQUATION OF THE SECOND ORDER IN HILBERT SPACE

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**ABSTRACT.** We consider nonlinear perturbations of the hyperbolic equation in the Hilbert space. Necessary and sufficient conditions for the existence of solutions of boundary-value problem for the corresponding equation and iterative procedures for their finding are obtained in the case when the operator in linear part of the problem hasn't inverse and can have nonclosed set of values. As an application we consider boundary-value problem for van der Pol equation in a separable Hilbert space.

### 1. Introduction

It is well-known that hyperbolic systems of equations are a class of wave equations that can be used to describe various processes in nature. great number of fundamental results have been obtained in this area [20], [14], [28], [27], [12], [19]. Nevertheless, research on various aspects of the qualitative theory of such equations remains relevant [25], [2], [9], [24], [21], [10], [11], [23]. Such questions include, in particular, the study of the existence and construction of

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2020 *Mathematics Subject Classification.* Primary: 34G20, 35L10; Secondary: 35L70, 35L90.

*Key words and phrases.* Boundary-value problem; nonlinear hyperbolic differential equation; van der Pol equation; Moore–Penrose pseudo-inverse matrix..

The author acknowledges the financial support from the grant “Boundary-Value Problems and Impulse Perturbations of Nonlinear Evolution Equations in Infinite-Dimensional Spaces” 3M2022 (Reg. No. 0122U002463) and the SOMPATY Project (European Union’s 2 Horizon 2020 Research and Innovation Programme under the Marie Skłodowska-Curie Grant No. 873071).