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REVERSE FABER–KRAHN INEQUALITIES FOR ZAREMBA PROBLEMS

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Abstract. Let Ω be a domain in $\mathbb{R}^n$ $(n \geq 2)$ of the form $\Omega = \Omega_{\rm out} \setminus \overline{\Omega_{\rm in}}$. Set Ω_D to be either $\Omega_{\rm out}$ or $\Omega_{\rm in}$. For $p \in (1, \infty)$, and $q \in [1, p]$, let $\tau_{1,q}(\Omega)$ be the first eigenvalue of

$$\begin{aligned} -\Delta_p u &= \tau \left(\int_\Omega |u|^q \, dx \right)^{(p-q)/q} |u|^{q-2} u && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega_D, \\ \frac{\partial u}{\partial \eta} &= 0 && \text{on } \partial\Omega \setminus \partial\Omega_D. \end{aligned}$$

Under the assumption that Ω_D is convex, we establish the following reverse Faber–Krahn inequality

$$\tau_{1,q}(\Omega) \leq \tau_{1,q}(\Omega^\star),$$

where $\Omega^\star = B_R \setminus \overline{B_r}$ is a concentric annular region in $\mathbb{R}^n$ having the same Lebesgue measure as Ω and such that

(i) (when $\Omega_D = \Omega_{\rm out}$) $W_1(\Omega_D) = \omega_n R^{n-1}$, and $(\Omega^\star)_D = B_R$,
(ii) (when $\Omega_D = \Omega_{\rm in}$) $W_{n-1}(\Omega_D) = \omega_n r$, and $(\Omega^\star)_D = B_r$.

Here $W_i(\Omega_D)$ is the $i^{\rm th}$ *quermassintegral* of Ω_D. We also establish Sz.-Nagy's type inequalities for parallel sets of a convex domain in $\mathbb{R}^n$ $(n \geq 3)$ for our proof.

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