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BMO AND DIRICHLET PROBLEM FOR SEMI-LINEAR EQUATIONS IN THE PLANE

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Abstract. First we study the Dirichlet problem $\operatorname{Re}\omega(z) \to \varphi(\zeta)$ as $z \to \zeta$, $z \in D, \zeta \in \partial D$, with continuous boundary data $\varphi\colon \partial D \to \mathbb{R}$ for semi-linear Beltrami equations $\omega_{\overline{z}} - \mu(z)\omega_z = \sigma(z)q(\operatorname{Re}\omega(z))$. We assume here that D is an arbitrary bounded domain of the complex plane $\mathbb{C}$, which is either simply connected or has no boundary components degenerated to a single point, and that the equations are locally uniform elliptic with possible singularities at the boundary. For $\sigma \in L_p(D)$, $p > 2$, with compact support, and continuous $q\colon \mathbb{R} \to \mathbb{C}$, $q(t)/t \to 0$ as $t \to \infty$, we establish a series of effective criteria for existence of solutions of the Dirichlet problem in terms of BMO, FMO, Calderon-Zygmund, Lehto and Orlicz integral means. We also establish representation and regularity of these solutions. Then, we prove existence, representation and regularity of weak solutions of the Dirichlet problem $u(z) \to \varphi(\zeta)$ as $z \to \zeta$, $z \in D, \zeta \in \partial D$, to semi-linear Poisson type equations $\operatorname{div}[A(z)\nabla u(z)] = g(z)\,Q(u(z))$ for $g \in L_p(D)$, $p > 1$, with compact support, and continuous $Q : \mathbb{R} \to \mathbb{R}$, $Q(t)/t \to 0$ as $t \to \infty$. We also assume here conditions on the matrix coefficients $A(z)$ guaranteing locally uniform ellipticity of these equations. Finally, we give examples of possible applications of the obtained results to various semi-linear equations of the mathematical physics in anisotropic and inhomogeneous media.

2020 *Mathematics Subject Classification.* Primary: 30C62, 30C65, 30E25; Secondary: 30G30, 35F45, 35J25.

Key words and phrases. BMO, bounded mean oscillation; FMO, finite mean oscillation; Dirichlet problem; semi-linear Beltrami equations; semi-linear Poisson type equations; generalized analytic functions with sources, generalized harmonic functions with sources.

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