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NEUMANN-TYPE BOUNDARY VALUE PROBLEM ASSOCIATED WITH HAMILTONIAN SYSTEMS

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Abstract. The aim of this paper is to investigate some multiplicity results for a Hamiltonian system with Neumann-type boundary conditions. A critical point theory is applied in order to show that the the problem has multiple solutions. The crucial part of this paper is that, in contrast to periodic problems where the Poincaré–Birkhoff Theorem has a significant role, no twist condition is required.

1. Introduction

Hamiltonian systems are one of the most popular dynamical systems in which researchers are engaged with. Some of them found periodic solutions using critical point theory, topological degree, and the Poincaré–Birkhoff Theorem [2]–[4], [9]–[13]. Some others are investigating the existence of multiple solutions to Hamiltonian systems with different boundary conditions [1], [6], [7].

Recently Fonda and Ortega in [7] proved the existence of at least $N+1$ geometrically distinct solutions to the problem

$$\begin{cases} x' = \nabla_y H(t,x,y), \quad y' = -\nabla_x H(t,x,y), \\ y(a) = 0 = y(b), \end{cases} \tag{1.1}$$

using a critical point theorem of Szulkin [15], based on an infinite-dimensional extension of the classical Lusternik–Schnirelmann theory on the multiplicity of

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