

MINIMIZERS OF L^2 -CRITICAL INHOMOGENEOUS VARIATIONAL PROBLEMS WITH A SPATIALLY DECAYING NONLINEARITY IN BOUNDED DOMAINS

HONGFEI ZHANG — SHU ZHANG

ABSTRACT. We consider the minimizers of L^2 -critical inhomogeneous variational problems with a spatially decaying nonlinear term in an open bounded domain Ω of \mathbb{R}^N which contains 0. We prove that there is a threshold $a^* > 0$ such that minimizers exist for $0 < a < a^*$ and the minimizer does not exist for any $a > a^*$. In contrast to the homogeneous case, we show that both the existence and nonexistence of minimizers may occur at the threshold a^* depending on the value of $V(0)$, where $V(x)$ denotes the trapping potential. Moreover, under some suitable assumptions on $V(x)$, based on a detailed analysis on the concentration behavior of minimizers as $a \nearrow a^*$, we prove local uniqueness of minimizers when a is close enough to a^* .

1. Introduction

In this paper, we consider the following L^2 -critical constraint inhomogeneous variational problem

$$(1.1) \quad e(a) := \inf_{u \in \mathcal{M}} E_a(u),$$

2020 *Mathematics Subject Classification*. Primary: 35Q40, 46N50; Secondary: 35J20, 35J60.

Key words and phrases. L^2 -critical; spatially decaying nonlinear; minimizers; concentration behavior; local uniqueness.