SECTIONAL CATEGORY
OF MAPS RELATED TO FINITE SPACES

KOHEI TANAKA

ABSTRACT. In this study, we compute some examples of sectional category $\text{secat}(f)$ and sectional number $\text{sec}(f)$ for continuous maps $f$ related to finite spaces. Moreover, we introduce an invariant $\text{secat}_k(f)$ for a map $f$ between finite spaces using the $k$-th barycentric subdivision and show the equality $\text{secat}_k(f) = \text{sec}(B(f))$ for sufficiently large $k$, where $B(f)$ is the induced map on the associated polyhedra.

1. Introduction

The sectional category $\text{secat}(f)$ of a continuous map $f : X \to Y$ between topological spaces $X$ and $Y$ is a numerical invariant originally introduced in [16]. This is defined as the smallest number $n$ such that there exist $n + 1$ open sets covering $Y$, where each open set admits a homotopy section of $f$. Several numerical invariants of topological spaces are expressed as the sectional category of special maps.

The Lusternik–Schnirelmann (LS) category $\text{cat}(X)$ of a space $X$ is the smallest number $n$ such that there exist $n + 1$ categorical open sets covering $X$ (see [11]). Here, a subset $U \subset X$ is categorical if $U$ is contractible in $X$; that is, the inclusion $U \hookrightarrow X$ is null-homotopic. The LS category $\text{cat}(X)$ agrees with the sectional category $\text{secat}(f)$ of a null-homotopic map to $X$. Furthermore, the

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