NORMALIZED SOLUTIONS FOR THE SCHRÖDINGER–POISSON SYSTEM WITH DOUBLY CRITICAL GROWTH

YUXI MENG — XIAOMING HE

Abstract. In this paper we are concerned with normalized solutions to the Schrödinger–Poisson system with doubly critical growth

\[
\begin{aligned}
-\Delta u - \phi |u|^3 u &= \lambda u + \mu |u|^{q-2} u + |u|^4 u, \quad x \in \mathbb{R}^3, \\
-\Delta \phi &= |u|^5, \quad x \in \mathbb{R}^3,
\end{aligned}
\]

and prescribed mass

\[
\int_{\mathbb{R}^3} |u|^2 \, dx = a^2,
\]

where \(a > 0\) is a constant, \(\mu > 0\) is a parameter and \(2 < q < 6\). In the \(L^2\)-subcritical case, we study the multiplicity of normalized solutions by applying the truncation technique, and the genus theory; and in the \(L^2\)-supercritical case, we obtain a couple of normalized solutions by developing a fiber map. Under both cases, to recover the loss of compactness of the energy functional caused by the critical growth, we need to adopt the concentration-compactness principle. Our results complement and improve some related studies for the Schrödinger–Poisson system with nonlocal critical term in the literature.