Topological **M**ethods in **N**onlinear **A**nalysis
DOI: 10.12775/TMNA.2024.008

TOPOLOGIZING SPERNER'S LEMMA

Przemysław Tkacz

Abstract. The aim of this paper is to extend Sperner's lemma for a new class of complexes, called n-Sperner. Next, we consider a topological version of Sperner's lemma, which leads to characterization of the covering dimension and KKM-principle. Finally, for an arbitrary topological space a new dimension is defined.

1. Introduction

The classical Sperner lemma [16] has been generalized in a variety of ways. Some authors [2], [8] considered multilabeled versions i.e. each vertex assigns few labels, the others [1], [5], [12] studied colorful one, here the number of labels can be much greater than the dimension of the simplicial complex. Finally, in some extensions n-dimensional simplices are replaced by wider class of complexes as a polytopal bodies [12] or PL-manifolds [13]. In this paper we present a generalization of Sperner's lemma for a new type of complexes (polyhedra), called n-Sperner. Such objects do not seem to have been defined before, because this class contains examples of not convex, not connected, not strong connected as well as not pure complexes, no assumption about being (pseudo)manifold or having fixed-point property is needed. That is demonstrated rather strikingly by simple examples of the n-Sperner polyhedra, presented in Section 3 (especially see Figure 1). It is worth pointing out that the conditions under which Sperner's lemma holds for simplicial complexes of dimension n are given in [4,

2020 *Mathematics Subject Classification.* 54F45, 54H25, 52B05, 52B11, 05C38.
Key words and phrases. dimension; labeling; KKM principle; simplicial complex; Sperner's lemma.

Please cite this article as: Przemysław Tkacz, *Topologizing Sperner's lemma*, Topol. Methods Nonlinear Anal. (2024).
DOI: 10.12775/TMNA.2024.008