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GLOBAL EXISTENCE, LOCAL EXISTENCE AND BLOW-UP OF MILD SOLUTIONS FOR ABSTRACT TIME-SPACE FRACTIONAL DIFFUSION EQUATIONS

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ABSTRACT. In this paper, we consider initial boundary value problems for abstract fractional diffusion equations $\partial_t^\beta u + (-\Delta)^s u = g(t, x, u)$ with the Caputo time fractional derivatives and fractional Laplacian operators. When $g(t, x, u)$ satisfies condition (G), problems can be applied by a strong maximum principle involving time-space fractional derivatives. Hence, we establish the global existence and uniqueness of mild solution by upper and lower solutions method. Moreover, the mild solution mentioned above turns out to be a classical solution. When condition (G) does not hold, then we study nonexistence of global solutions under certain conditions, and we obtain the local existence and blow-up of mild solutions. Further, we conclude that the first eigenvalue λ_1 seems to be a critical value for nonlinear problems.

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Key words and phrases. Global existence; mild solution; upper and lower solutions; blow up; time-space fractional derivatives.

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