

## EXACT CONTROLLABILITY OF INFINITE DIMENSIONAL SYSTEMS WITH CONTROLS OF MINIMAL NORM

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*Dedicated to the memory of Professor Ioan I. Vrabie*

ABSTRACT. The paper deals with the exact controllability of a semilinear system in a separable Hilbert space. A bounded linear part is considered and a linear control introduced. The state space is compactly embedded in a Banach space and the nonlinear term is continuous in its state variable in the norm of the Banach space. An infinite sequence of finite dimensional controllability problems is introduced and the solution is obtained by a limiting procedure. To the best of our knowledge, the method is new in controllability theory. An application to an integro-differential system in euclidean spaces completes the discussion.

### 1. Introduction

This paper deals with the exact controllability in infinite dimensional spaces, by means of linear controls. We consider the semilinear equation

$$(1.1) \quad y'(t) = Ay(t) + f(t, y(t)) + Bu(t), \quad t \in [0, T], \quad y(t) \in H,$$

with  $0 < T < +\infty$ , in the separable Hilbert space  $H$  and assume that the control term  $u$  belongs to  $L^2([0, T], U)$  where  $U$  is a Hilbert space. The operators  $A: H \rightarrow H$  and  $B: U \rightarrow H$  are linear and bounded. We refer to Section 3 for

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