REMARKS ON SOME LIMITS APPEARING IN THE THEORY OF ALMOST PERIODIC FUNCTIONS

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ABSTRACT. In this note we are going to present new short proofs concerning either the existence or the non-existence of some limits appearing in the theory of almost periodic functions. Our proofs are completely different from those presented in the papers [1] and [3].

1. Introduction

In the rich theory of almost periodic functions (see e.g. [6]) problems concerning the evaluation of the limit

$$\lim_{x \to +\infty} \frac{f(x)}{2 + \cos x + \cos(x\sqrt{2})},$$

where $f: \mathbb{R} \to \mathbb{R}$ is an exponential function or a polynomial (cf. [1] or [3]), quite frequently appear. This is connected to the fact that the function

$$x \mapsto \frac{1}{2 + \cos x + \cos(x\sqrt{2})} \quad \text{for } x \in \mathbb{R}$$

constitutes a classical example of a function which is either almost periodic in the sense of Levitan (briefly: LAP) or almost periodic with respect to the Lebesgue measure (briefly: µ.a.p.) (see e.g. [4]). In particular, in [1] the authors used the theory of continued fractions to prove that the limit (1.1) is equal to zero if

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