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CONVERGENCE ESTIMATES FOR ABSTRACT SECOND ORDER DIFFERENTIAL EQUATIONS WITH TWO SMALL PARAMETERS AND MONOTONE NONLINEARITIES

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ABSTRACT. In a real Hilbert space H we consider the following perturbed Cauchy problem

$$(P_{\varepsilon\delta}) \quad \begin{cases} \varepsilon u''_{\varepsilon\delta}(t) + \delta u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) + B(u_{\varepsilon\delta}(t)) = f(t), & t \in (0, T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases}$$

where $u_0, u_1 \in H$, $f: [0, T] \mapsto H$ and ε, δ are two small parameters, A is a linear self-adjoint operator, B is a locally Lipschitz and monotone operator. We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(P_{\varepsilon\delta})$ in two different cases:

- (i) when $\varepsilon \rightarrow 0$ and $\delta \geq \delta_0 > 0$;
- (ii) when $\varepsilon \rightarrow 0$ and $\delta \rightarrow 0$.

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighborhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

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