TOPOLOGICALLY ANOSOV PLANE HOMEOMORPHISMS

GONZALO COUSILLAS — JORGE GROIzman — JULIANA XAVIER

Abstract. This paper deals with classifying the dynamics of topologically Anosov plane homeomorphisms. We prove that a topologically Anosov homeomorphism $f: \mathbb{R}^2 \to \mathbb{R}^2$ is conjugate to a homothety if it is the time one map of a flow. We also obtain results for the cases when the nonwandering set of $f$ reduces to a fixed point, or if there exists an open, connected, simply connected proper subset $U$ such that $f(U) \subset \text{Int}(U)$, and such that

$$\bigcup_{n \leq 0} f^n(U) = \mathbb{R}^2.$$

In the general case, we prove a structure theorem for the $\alpha$-limits of orbits with empty $\omega$-limit (or the $\omega$-limits of orbits with empty $\alpha$-limit).

1. Introduction

A homeomorphism $f: M \to M$ of the metric space to itself is called expansive if there exists $\alpha > 0$ such that given $x, y \in M, x \neq y$, then $d(f^n(x), f^n(y)) > \alpha$ for some $n \in \mathbb{Z}$. The number $\alpha$ is called the expansivity constant of $f$.

The study of expansive systems is both classic and fascinating. In Lewowicz’s words [10], the fact that every point has a distinctive dynamical meaning implies that a rich interaction between dynamics and topology is to be expected.

If $\delta > 0$, a $\delta$-pseudo-orbit for $f$ is a sequence $(x_n)_{n \in \mathbb{Z}}$ such that $d(f(x_n), x_{n+1})$ is less than $\delta$ for all $n \in \mathbb{Z}$. If $\varepsilon > 0$, we say that the orbit of $x$ $\varepsilon$-shadows a given pseudo-orbit if $d(x_n, f^n(x)) < \varepsilon$ for all $n \in \mathbb{Z}$. Finally, we say that $f$ has the

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