THE CONTINUITY
OF ADDITIVE AND CONVEX FUNCTIONS
WHICH ARE UPPER BOUNDED
ON NON-FLAT CONTINUA IN $\mathbb{R}^n$

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ABSTRACT. We prove that for a continuum $K \subset \mathbb{R}^n$ the sum $K^{+n}$ of $n$ copies of $K$ has non-empty interior in $\mathbb{R}^n$ if and only if $K$ is not flat in the sense that the affine hull of $K$ coincides with $\mathbb{R}^n$. Moreover, if $K$ is locally connected and each non-empty open subset of $K$ is not flat, then for any (analytic) non-meager subset $A \subset K$ the sum $A^{+n}$ of $n$ copies of $A$ is not meager in $\mathbb{R}^n$ (and then the sum $A^{+2n}$ of $2n$ copies of the analytic set $A$ has non-empty interior in $\mathbb{R}^n$ and the set $(A - A)^{+n}$ is a neighbourhood of zero in $\mathbb{R}^n$). This implies that a mid-convex function $f : D \to \mathbb{R}$ defined on an open convex subset $D \subset \mathbb{R}^n$ is continuous if it is upper bounded on some non-flat continuum in $D$ or on a non-meager analytic subset of a locally connected nowhere flat subset of $D$.

1. Introduction

Let $X$ be a linear topological space over the field of real numbers. A function $f : X \to \mathbb{R}$ is called additive if $f(x + y) = f(x) + f(y)$ for all $x, y \in X$.

A function $f : D \to \mathbb{R}$ defined on a convex subset $D \subset X$ is called mid-convex if $f((x + y)/2) \leq (f(x) + f(y))/2$ for all $x, y \in D$.

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