Topological Methods in Nonlinear Analysis
DOI: 10.12775/TMNA.2019.037

PERIODIC SOLUTIONS FOR A SINGULAR LIÉNARD EQUATION WITH INDEFINITE WEIGHT

Shiping Lu — Runyu Xue

Abstract. In this paper, the existence of positive periodic solutions is studied for a singular Liénard equation where the weight function has an indefinite sign. Due to the lack of a priori estimates over the set of all possible positive periodic solutions in this equation, a new method is proposed for estimating a priori bounds of positive periodic solutions. By the use of a continuation theorem of the Mawhin coincidence degree, new conditions for existence of positive periodic solutions to the equation are obtained. The main results show that the singularity of coefficient function associated to the friction term at $x = 0$ may help the existence of periodic solutions.

1. Introduction

The purpose of this paper is to study the existence of positive T-periodic solutions for a singular Liénard equation with indefinite weight

$$x''(t) + f(x(t))x'(t) + \frac{\alpha(t)}{x^\mu(t)} = h(t), \tag{1.1}$$

where $f \in C((0,+\infty),\mathbb{R})$ may have a singularity at $x = 0$, $\mu \in (0,+\infty)$ is a constant, α and h are T-periodic functions with $\alpha, h \in L^1([0,T],\mathbb{R})$. Since the weight function α may change sign on $[0,T]$, the singularity of the term $\alpha(t)/x^\mu$

2010 *Mathematics Subject Classification.* Primary: 34B16, 34K13, 34C25.
Key words and phrases. Liénard equation; Mawhin's continuation theorem; periodic solution; singularity.
The first author was supported in part by NSF of China Grant No. 11271197.

Please cite this article as: Shiping Lu, Runyu Xue, *Periodic solutions for a singular Liénard equation with indefinite weight*, Topol. Methods Nonlinear Anal. (2019).
DOI: 10.12775/TMNA.2019.037