

KRASNOSEL'SKIĀ-SCHAEFER TYPE METHOD IN THE EXISTENCE PROBLEMS

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ABSTRACT. We consider a general integral equation satisfying algebraic conditions in a Banach space. Using Krasnosel'skiĀ-Schaefer type method and technical assumptions, we prove an existence theorem producing a periodic solution of some nonlinear integral equation.

1. Introduction and preliminaries

Let $(\mathcal{B}, \|\cdot\|)$ be the Banach space of continuous Γ -periodic functions $\varphi: \mathbb{R} \rightarrow \mathbb{R}$ with $\Gamma > 0$ and the supremum norm. In this paper, we study the following integral equation

$$(1.1) \quad \varphi(t) = f(t, \varphi(t)) - \int_{t-\alpha}^t D(t, s)g(s, \varphi(s)) ds,$$

where $\alpha > 0$, $f, g, D: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions satisfying the following assumptions:

- (A1) $f(t + \Gamma, x) = f(t, x)$, $D(t + \Gamma, s + \Gamma) = D(t, s)$, $g(t + \Gamma, x) = g(t, x)$ for all $s, t, x \in \mathbb{R}$,
- (A2) $D(t, t - \alpha) = 0$, $D_{st}(t, s) \leq 0$, $D_s(t, s) \geq 0$ for all $t \in \mathbb{R}$ and $s \in (t - \alpha, t)$,
- (A3) the function D_{st} is continuous,

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