A WEIGHTED TRUDINGER–MOSER TYPE INEQUALITY 
AND ITS APPLICATIONS 
TO QUASILINEAR ELLIPTIC PROBLEMS 
WITH CRITICAL GROWTH 
IN THE WHOLE EUCLIDEAN SPACE

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ABSTRACT. We establish a version of the Trudinger-Moser inequality involving unbounded or decaying radial weights in weighted Sobolev spaces. In the light of this inequality and using a minimax procedure we also study existence of solutions for a class of quasilinear elliptic problems involving exponential critical growth.

1. Introduction and main results

We recall that if $\Omega$ is a bounded domain in $\mathbb{R}^n$ ($n \geq 2$), the classical Trudinger–Moser inequality (cf. [31], [38]) asserts that $e^{\alpha |u|^n} \in L^1(\Omega)$, for all $u \in W^{1,n}_0(\Omega)$ and $\alpha > 0$ and there exists a constant $C(n) > 0$ such that

$$\sup_{\|u\|_{n'} \leq 1} \int_{\Omega} e^{\alpha |u|^n} \, dx \leq C(n)|\Omega|, \quad \text{if } \alpha \leq \alpha_n,$$

where $n' = n/(n-1)$, $\alpha_n = n\omega_{n-1}^{1/(n-1)}$, $\|u\|_n := (\int_{\Omega} |\nabla u|^n \, dx)^{1/n}$ and $\omega_{n-1}$ is the surface area of the unit sphere in $\mathbb{R}^n$. Moreover, the inequality (1.1)

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