CLASSIFICATION OF RADIAL SOLUTIONS
TO HÉNON TYPE EQUATION
ON THE HYPERBOLIC SPACE

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ABSTRACT. We devote this paper to classifying radial solutions of a weighted semilinear elliptic equation on the hyperbolic space. More precisely, for a weighted Lane–Emden equation on the hyperbolic space, we shall study the sign and asymptotic behavior of the radial solutions. We shall also show the existence of fast-decay sign-changing solutions to the Lane–Emden equation on the hyperbolic space.

1. Introduction

In this paper, we shall investigate the structure of radial solutions to the following weighted semilinear elliptic equation:

\[ -\Delta_g u = (\sinh r)^\alpha |u|^{p-1} u \quad \text{in } \mathbb{H}^N, \]

where \( N \geq 2, \ p > 1, \) and \( \alpha > -2. \) Here, \( \mathbb{H}^N \) denotes the \( N \)-dimensional hyperbolic space in terms of the spherical coordinates, \( r > 0 \) represents the geodesic distance on \( \mathbb{H}^N, \) and \( \Delta_g \) denotes the Laplace–Beltrami operator on \( \mathbb{H}^N. \)

The structure of radial solutions to semilinear elliptic equations has attracted a great interest. In particular, the following Hénon type equation has been well

\[ \begin{align*}
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\end{align*} \]

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