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HETEROCLINIC SOLUTIONS OF ALLEN–CAHN TYPE EQUATIONS WITH A GENERAL ELLIPTIC OPERATOR

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Abstract. We consider a generalization of the Allen–Cahn type equation in divergence form $-\mathrm{div}(\nabla G(\nabla u(x,y))) + F_u(x,y,u(x,y)) = 0$. This is more general than the usual Laplace operator. We prove the existence and regularity of heteroclinic solutions under standard ellipticity and m-growth conditions.

1. Introduction

The Allen–Cahn equation is a well-known elliptic partial differential equation considered by many authors in the form:

$$-\Delta u(x,y) + F_u(x,y,u) = 0$$

where F is a double-well potential of u and has some other standard properties like periodicity in x and y (see the next section for details). Here we are not interested in the Dirichlet problem but in the existence of heteroclinic solutions in the whole of $\mathbb{R}^2$. This problem was widely studied and there are many articles that contain the existence theorems about such solutions. As an example we can take [11] where the authors show the existence and multiplicity of heteroclinic and some other special types of solutions. Earlier in [1] and [2] the problem was solved in a more simple form where $F(x,u) = f(x)F(u)$.

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