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CONVENIENT MAPS FROM ONE-RELATOR MODEL TWO-COMPLEXES INTO THE REAL PROJECTIVE PLANE

MARCIO COLOMBO FENILLE

ABSTRACT. Let f be a map from a one-relator model two-complex $K_{\mathcal{P}}$ into the real projective plane. The composition $\varrho \circ f_{\#}$ of the homomorphism $f_{\#}$ induced by f on fundamental groups with the action ϱ of $\pi_1(\mathbb{RP}^2)$ over $\pi_2(\mathbb{RP}^2)$ provides a local integer coefficient system $f_{\#}^{\varrho}$ over $K_{\mathcal{P}}$. We prove that if the twisted integer cohomology group $H^2(K_{\mathcal{P}};{}_{f_{\#}^{\varrho}}\mathbb{Z}) = 0$, then f is homotopic to a non-surjective map. As an intermediary step for the proof, we show that if $H^2(K_{\mathcal{P}};{}_{\beta}\mathbb{Z}) = 0$ for some local integer coefficient system β over $K_{\mathcal{P}}$, then $K_{\mathcal{P}}$ is aspherical.

1. Introduction

The existence of strong surjections from a finite and connected n-dimensional CW complex K (a n-complex, to shorten) into a closed n-manifold Y has been investigated for at least a decade, specially from the viewpoint of the topological root theory.

For a *strong surjection* from K into Y we mean a (continuous) map $f\colon K \to Y$ whose free homotopy class $[f] \in [K; Y]$ has just surjective maps. In this case, we say also that f is *strongly surjective*. In the context of topological root theory, a map $f\colon K \to Y$ which is not strongly surjective is said to be *root free*.

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