

## EXISTENCE, UNIQUENESS AND PROPERTIES OF GLOBAL WEAK SOLUTIONS TO INTERDIFFUSION WITH VEGARD RULE

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**ABSTRACT.** We consider the diffusional transport in an  $r$ -component solid solution. The model is expressed by the nonlinear system of strongly coupled parabolic differential equations with initial and nonlinear boundary conditions. The techniques involved are the local mass conservation law for fluxes, which are a sum of the diffusional and Darken drift terms, and the Vegard rule. The considered boundary conditions allow the physical system to be not only closed but also open. The theorems on existence, uniqueness and properties of global weak solutions are proved. The main tool used in the proof of the existence result is the Galerkin approximation method. The agreement between the theoretical results, numerical simulations and experimental data is shown.

### 1. Introduction

Quantitative description of the diffuse mass transport is particularly essential for materials processing and hydrodynamics. It is important for the Navier–Stokes problem, where it allows considering diffusion in multicomponent fluids [15]. An inspiring effort dedicated to the rigorous mathematical treatment

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*Key words and phrases.* Interdiffusion; Darken method; Vegard rule; parabolic nonlinear system; existence; uniqueness; properties of global weak solutions; Galerkin approximation.

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