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EXISTENCE AND EXPONENTIAL STABILITY OF ANTI-PERIODIC SOLUTION FOR FUZZY BAM NEURAL NETWORKS WITH INERTIAL TERMS AND TIME-VARYING DELAYS

Yongkun Li — Jiali Qin

Abstract. In this paper, the existence and exponential stability of anti-periodic solutions for fuzzy BAM neural networks with inertial terms and time-varying delays is investigated. Firstly, some sufficient conditions ensuring the existence of anti-periodic solutions of the system are obtained by employing a new continuation theorem of coincidence degree theory. Secondly, by constructing an appropriate Lyapunov function, some sufficient conditions are derived to guarantee the global exponential stability of anti-periodic solutions of the system. Our results of this paper are completely new. Finally, two numerical examples are given to show the effectiveness of the obtained results.

1. Introduction

Bidirectional associative memory (BAM) neural networks, which were first proposed by Kosko in 1987 [13], [14], consist of two layers of neurons arranged: the U-layer and the V-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there is no interconnection among neurons

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in the same layer. BAM neural networks have many excellent characteristics because of their special structure with two layers of neurons interconnected. Compared with the neural networks with single-layer auto-associative correlation, they have more advantages in signal processing and image processing, combinatorial optimization, automatic control, pattern recognition and so on. As we know that time delays in interactions between neurons frequently exist, which may cause or destroy oscillation, instability and bifurcation to networks [16], [26] and [12]. Therefore, many researchers have studied the dynamics of BAM neural networks with various kinds of time delays such as the stability of equilibriums [17], the periodicity [18], the anti-periodicity [22], [32], the weighted pseudo-almost periodicity [3], the Hopf bifurcation analysis [30] and so on.

On the one hand, fuzzy cellular neural networks (FCNNs), which are a new type of cellular neural networks first introduced by Yang and Yang [34], combine the operations of fuzzy AND and fuzzy OR with cellular neural networks. It has been reported that FCNNs have their potential in image processing, pattern recognition [8], [15] and even in white blood cell detection [31]. FCNNs with their various variant models have been widely studied by many scholars and many results about the dynamics of them have been obtained. We refer the reader to [9], [29], [36], [10], [35] and the references cited therein. For a class of BAM neural networks combined with fuzzy operations, i.e. fuzzy BAM neural networks, there are also many results have been obtained [21], [37], [35], [20]. For example, the authors of [35] investigated the stability and lag synchronization for memristor-based fuzzy Cohen–Grossberg bidirectional associative memory (BAM) neural networks with mixed delays and impulses by employing the inequality technique, homeomorphism theory and constructing some suitable Lyapunov–Krasovskiĭ functionals; the authors of [20] studied discrete-time fuzzy BAM neural networks with variable delays and impulses, based on M-matrix theory and analytic methods, they established the existence and global exponential stability of a unique equilibrium and estimated the exponential convergence rate index.

On the other hand, incorporating inertial terms into neural networks is a very important approach for generating complex dynamical behaviors in neural networks [5]–[7], and there are also evident biological backgrounds for introducing inertial terms into the standard neural network models (see [2], [4]).

In addition, it is a very important topic that the existence and stability of anti-periodic solutions for differential equations. Okochi [25] first studied the anti-periodic solution of nonlinear evolution equations. The existence and stability of anti-periodic solutions play a key role in characterizing the behavior of nonlinear differential equations. Besides, it has been proved that the signal transmission process of neural networks can often be described as an anti-periodic process. Therefore, in recent years, the study on anti-periodic problems of neural networks has attracted much attention by many authors (see [22], [32], [37], [28], [19], [24], [33] and the references cited therein). Recently, some scholars have studied dynamics of inertial BAM neural networks [33], [27], [11], [38], [23]. For example, in [33], by applying inequality techniques and Lyapunov methods, sufficient conditions for the existence and exponential stability of anti-periodic solutions for a class of inertial BAM neural networks were obtained; in [11], by constructing a suitable Lyapunov function, using Weierstrass criteria and boundedness of solutions, the existence and exponential stability of periodic solutions for inertial BAM neural networks was studied; in [38], by using homeomorphism theory and constructing a suitable Lyapunov functional together with matrix equations, some delay-dependent sufficient conditions were established for ensuring the existence and global asymptotic stability of a class of neutral-type inertial BAM neural networks; in [23], by combining Mawhin's continuation theorem of coincidence degree theory with Lyapunov functional method, sufficient conditions for the existence of periodic solutions of inertial BAM neural networks were obtained and sufficient conditions ensuring the global asymptotic stability of periodic solutions of the system were obtained by using Lyapunov functional method and inequality techniques. However, up to now, there is no paper published on the dynamics of fuzzy BAM neural networks with inertial terms and time-varying delays.

Motivated by the aforementioned discussions, in this paper, we are concerned with the following fuzzy BAM neural network with inertial terms and time-varying delays:

$$\tag{1.1} \begin{cases} u_i''(t) = -\alpha_i u_i'(t) - a_i u_i(t) + \sum\limits_{j=1}^{m} c_{ij}(t) f_j(v_j(t)) \\ \qquad + \bigwedge\limits_{j=1}^{m} d_{ij}(t) f_j(v_j(t-\tau_{ji}(t))) \\ \qquad + \bigvee\limits_{j=1}^{m} e_{ij}(t) f_j(v_j(t-\tau_{ji}(t))) + I_i(t), \qquad i = 1, \dots, n, \\ \\ v_j''(t) = -\beta_j v_j'(t) - b_j v_j(t) + \sum\limits_{i=1}^{n} r_{ji}(t) g_i(u_i(t)) \\ \qquad + \bigwedge\limits_{i=1}^{n} p_{ji}(t) g_i(u_i(t-\sigma_{ij}(t))) \\ \qquad + \bigvee\limits_{i=1}^{n} q_{ji}(t) g_i(u_i(t-\sigma_{ij}(t))) + J_j(t), \qquad j = 1, \dots, m, \end{cases}$$

where the second derivative is called an inertial term of system (1.1); $\alpha_i > 0$, $\beta_j > 0$ are two constants; $u_i(t)$ and $v_j(t)$ are the states of the ith neuron from the neural field F_U and the jth neuron from the neural field F_V at time t, respectively; $a_i > 0$, $b_j > 0$ denote the rate with which the ith neuron and the jth neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs, respectively; $c_{ij}(t)$, $r_{ji}(t)$ denote the connection weights at time t; $d_{ij}(t)$, $p_{ji}(t)$ and $e_{ij}(t)$, $q_{ji}(t)$ are elements of fuzzy feedbacks MIN and MAX templates at time t, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and OR operations, respectively; $I_i(t)$ and $J_j(t)$ are the external inputs at time t; $\tau_{ji}(t) \geq 0$ and $\sigma_{ij}(t) \geq 0$ are the axonal signal transmission delays; f_j and g_i are activation functions.

Our main purpose of this paper is by using a new continuation theorem of coincidence degree theory to establish the existence of anti-periodic solutions for system (1.1) and by constructing an appropriate Lyapunov function to study the global exponential stability of anti-periodic solutions of system (1.1). To the best of our knowledge, this is the first time to study the anti-periodic dynamics of fuzzy BAM neural network with inertial terms and time-varying delays. Our results of this paper are completely new and it is worth mentioning that the method used in this paper to study the existence of anti-periodic solutions is completely different from the methods mentioned in the previous literature.

The remaining part of this paper is organized as follows. In Section 2, we introduce some notations and definitions and state some preliminary results which are needed in later sections. In Section 3, we derive some sufficient conditions for the existence and exponential stability of anti-periodic solutions of (1.1). In Section 4, two numerical examples are given to show the effectiveness of the obtained results. In Section 5, we give a brief conclusion.

2. Preliminaries

In order to derive sufficient conditions for the existence of anti-periodic solutions of system (1.1) under the assumptions that the activation functions are bounded or satisfied Lipschitz conditions, respectively. We need the following assumptions:

(H$_1$) For $i = 1, \ldots, n$, $j = 1, \ldots, m$, $c_{ij}, d_{ij}, e_{ij}, r_{ji}, p_{ji}, q_{ji}, \tau_{ji}, \sigma_{ij}, I_i, J_j \in C(\mathbb{R}, \mathbb{R})$ and for $t, x, y \in \mathbb{R}$, there exists positive number ω such that:

$$\begin{aligned} \tau_{ji}(t+\omega) &= \tau_{ji}(t), & \sigma_{ij}(t+\omega) &= \sigma_{ij}(t), \\ I_i(t+\omega) &= -I_i(t), & J_j(t+\omega) &= -J_j(t), \\ c_{ij}(t+\omega)f_j(x) &= -c_{ij}(t)f_j(-x), & d_{ij}(t+\omega)f_j(x) &= -d_{ij}(t)f_j(-x), \\ e_{ij}(t+\omega)f_j(x) &= -e_{ij}(t)f_j(-x), & r_{ji}(t+\omega)g_i(y) &= -r_{ji}(t)g_i(-y), \\ p_{ji}(t+\omega)g_i(y) &= -p_{ji}(t)g_i(-y), & q_{ji}(t+\omega)g_i(y) &= -q_{ji}(t)g_i(-y). \end{aligned}$$

(H$_2$) For $i = 1, \dots, n$, $j = 1, \dots, m$, activation functions f_j and g_i satisfy Lipschitz condition, i.e. there exist constants $L_f > 0$ and $L_g > 0$ such that for all $x, y \in \mathbb{R}$,

$$|f_j(x) - f_j(y)| \leq L_f |x - y|, \qquad |g_i(x) - g_i(y)| \leq L_g |x - y|.$$

(H$_2'$) For $i = 1, \dots, n$, $j = 1, \dots, m$, activation functions f_j, g_i are bounded, i.e. there exist constants $N_f > 0$ and $N_g > 0$ such that

$$|f_j(x)| \leq N_f \quad \text{and} \quad |g_i(x)| \leq N_g \quad \text{for all } x \in \mathbb{R}.$$

Remark 2.1. From (H$_1$) and the definition of fuzzy operators, it is easy to see that the fuzzy terms in system (1.1) are anti-periodic, that is, (1.1) is an anti-periodic system.

For ease of exposition, we will adopt the following notation:

$$\begin{aligned}
c_{ij} &= \sup_{t \in \mathbb{R}} |c_{ij}(t)|, & d_{ij} &= \sup_{t \in \mathbb{R}} |d_{ij}(t)|, \\
e_{ij} &= \sup_{t \in \mathbb{R}} |e_{ij}(t)|, & I_i &= \sup_{t \in \mathbb{R}} |I_i(t)|, \\
r_{ji} &= \sup_{t \in \mathbb{R}} |r_{ji}(t)|, & p_{ji} &= \sup_{t \in \mathbb{R}} |p_{ji}(t)|, \\
q_{ji} &= \sup_{t \in \mathbb{R}} |q_{ji}(t)|, & J_j &= \sup_{t \in \mathbb{R}} |J_j(t)|, \\
\tau &= \max_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} \Big\{ \sup_{t \in \mathbb{R}} \tau_{ji}(t), \sup_{t \in \mathbb{R}} \sigma_{ij}(t) \Big\}, \\
\overline{L}_f &= \max_{1 \leq j \leq m} \{|f_j(0)|\}, & \overline{L}_g &= \max_{1 \leq i \leq n} \{|g_i(0)|\}.
\end{aligned}$$

The initial value of system (1.1) is given by

$$\begin{cases} u_i(s) = \varphi_{ui}(s), \quad u_i'(s) = \psi_{ui}(s), \quad s \in [-\tau, 0], \ i = 1, \dots, n, \\ v_j(s) = \varphi_{vj}(s), \quad v_j'(s) = \psi_{vj}(s), \quad s \in [-\tau, 0], \ j = 1, \dots, m, \end{cases} \tag{2.1}$$

where $\varphi_{ui}, \psi_{ui}, \varphi_{vj}, \psi_{vj} \in C([-\tau, 0], \mathbb{R})$. Taking the variable transformation:

$$\begin{cases} y_i(t) = u_i'(t) + u_i(t), \quad i = 1, \dots, n, \\ z_j(t) = v_j'(t) + v_j(t), \quad j = 1, \dots, m, \end{cases}$$

for $i = 1, \ldots, n$, $j = 1, \ldots, m$, (1.1) and (2.1) can be written as:

$$
\tag{2.2}
\begin{cases}
u_i'(t) = -u_i(t) + y_i(t) \triangleq F_i(u_i(t), y_i(t)), \\
y_i'(t) = -(a_i - \alpha_i + 1)u_i(t) - (\alpha_i - 1)y_i(t) + \displaystyle\sum_{j=1}^{m} c_{ij}(t) f_j(v_j(t)) \\
\qquad + \displaystyle\bigwedge_{j=1}^{m} d_{ij}(t) f_j(v_j(t - \tau_{ji}(t))) \\
\qquad + \displaystyle\bigvee_{j=1}^{m} e_{ij}(t) f_j(v_j(t - \tau_{ji}(t))) + I_i(t) \\
\quad \triangleq G_i(t, u_i(t), y_i(t), v(t)), \\
v_j'(t) = -v_j(t) + z_j(t) \triangleq U_j(v_j(t), z_j(t)), \\
z_j'(t) = -(b_j - \beta_j + 1)v_j(t) - (\beta_j - 1)z_j(t) + \displaystyle\sum_{i=1}^{n} r_{ji}(t) g_i(u_i(t)) \\
\qquad + \displaystyle\bigwedge_{i=1}^{n} p_{ji}(t) g_i(u_i(t - \sigma_{ij}(t))) \\
\qquad + \displaystyle\bigvee_{i=1}^{n} q_{ji}(t) g_i(u_i(t - \sigma_{ij}(t))) + J_j(t) \\
\quad \triangleq H_j(t, u(t), v_j(t), z_j(t))
\end{cases}
$$

with the initial value:

$$
\tag{2.3}
\begin{cases}
u_i(s) = \varphi_{ui}(s), \quad u_i'(s) = \psi_{ui}(s), & s \in [-\tau, 0], \\
y_i(s) = \varphi_{ui}(s) + \psi_{ui}(s) = \phi_{ui}(s), & s \in [-\tau, 0], \\
v_j(s) = \varphi_{vj}(s), \quad v_j'(s) = \psi_{vj}(s), & s \in [-\tau, 0], \\
z_j(s) = \varphi_{vj}(s) + \psi_{vj}(s) = \phi_{vj}(s), & s \in [-\tau, 0].
\end{cases}
$$

Remark 2.2. Obviously, if $x = (u, y, v, z)^T = (u_1, \ldots, u_n, y_1, \ldots, y_n, v_1, \ldots, v_m, z_1, \ldots, z_m)^T$ is a solution of (2.2), then $(u, v)^T = (u_1, \ldots, u_n, v_1, \ldots, v_m)^T$ is a solution of (1.1).

Lemma 2.3 ([34]). *Suppose u and v are two states of system* (1.1), *then*

$$
\bigg| \bigwedge_{j=1}^{n} d_{ij}(t) g_j(u) - \bigwedge_{j=1}^{n} d_{ij}(t) g_j(v) \bigg| \le \sum_{j=1}^{n} |d_{ij}(t)| |g_j(u) - g_j(v)|,
$$

$$
\bigg| \bigvee_{j=1}^{n} e_{ij}(t) g_j(u) - \bigvee_{j=1}^{n} e_{ij}(t) g_j(v) \bigg| \le \sum_{j=1}^{n} |e_{ij}(t)| |g_j(u) - g_j(v)|.
$$

Lemma 2.4 ([1]). *Let X and Y be Banach spaces, and let $L\colon \operatorname{Dom} L \subset X \to Y$ be linear, $N\colon X \to Y$ be continuous. Assume that L is one-to-one and $K := L^{-1}N$ is compact. Furthermore, assume that there exists a bounded and* *open subset* $\Omega \subset X$ *with* $0 \in \Omega$ *such that equation* $Lu = \lambda Nu$ *has no solutions in* $\partial\Omega \cap \operatorname{Dom} L$ *for any* $\lambda \in (0,1)$. *Then the equation* $Lu = Nu$ *has at least one solution in* $\overline{\Omega}$.

Lemma 2.5 (Wirtinger's Inequality, [1]). *If* u *is a* C^1 *function such that* $u(0) = u(T)$, *then*

$$\|u - \overline{u}\|_{L_2} \le \frac{T}{2\pi}\|u'\|_{L_2},$$

where

$$\|u\|_{L_2} := \left(\int_0^T |u(t)|^2\,dt\right)^{1/2} \quad \textit{and} \quad \overline{u} = \frac{1}{T}\int_0^T u(t)\,dt.$$

Definition 2.6. Let $x = (u_1,\dots,u_n,y_1,\dots,y_n,v_1,\dots,v_m,z_1,\dots,z_m)^T \in \mathbb{R}^{2(m+n)}$ and $x^* = (u_1^*,\dots,u_n^*,y_1^*,\dots,y_n^*,v_1^*,\dots,v_m^*,z_1^*,\dots,z_m^*)^T \in \mathbb{R}^{2(m+n)}$ be any two solutions of system (2.2) with initial values

$$\Phi = (\varphi_{u1},\dots,\varphi_{un},\phi_{y1},\dots,\phi_{yn},\varphi_{v1},\dots,\varphi_{vm},\phi_{z1},\dots,\phi_{zm})^T,$$
$$\Phi^* = (\varphi_{u1}^*,\dots,\varphi_{un}^*,\phi_{y1}^*,\dots,\phi_{yn}^*,\varphi_{v1}^*,\dots,\varphi_{vm}^*,\phi_{z1}^*,\dots,\phi_{zm}^*)^T,$$

respectively. If there exist constants $\varepsilon > 0$ and $M > 0$ which are independent of the given solutions x and x^* such that

$$\|x(t) - x^*(t)\| \le M\|\Phi - \Phi^*\|e^{-\varepsilon t}, \quad t > 0,$$

where

$$\begin{aligned}\|\Phi - \Phi^*\| = &\sum_{i=1}^n \left\{\sup_{s\in[-\tau,0]}(|\varphi_{ui}(s) - \varphi_{ui}^*(s)| + \sup_{s\in[-\tau,0]}|\phi_{yi}(s) - \phi_{yi}^*(s)|\right\}\\ &+ \sum_{j=1}^m \left\{\sup_{s\in[-\tau,0]}|\varphi_{vj}(s) - \varphi_{vj}^*(s)| + \sup_{s\in[-\tau,0]}|\phi_{zj}(s) - \phi_{zj}^*(s)|\right\}.\end{aligned}$$

Then every solution of system (2.2) is said to be globally exponentially stable.

3. Main results

In this section, we first establish the existence of anti-periodic solutions of (1.1) by using Lemma 2.4. Then we derive some sufficient conditions for the exponential stability of anti-periodic solutions of (1.1) by constructing a proper Lyapunov function. Set

$$\begin{aligned}\mathbb{X} = \big\{x = (u,y,v,z)^T = (u_1,\dots,u_n,y_1,\dots,y_n,v_1,\dots,v_m,z_1,\dots,z_m)^T\\ \text{in } C(\mathbb{R},\mathbb{R}^{2(m+n)}), x(t+\omega) = -x(t)\big\}\end{aligned}$$

and let

$$\|x\|_{\mathbb{X}} = \sum_{i=1}^n(|u_i|_\infty + |y_i|_\infty) + \sum_{j=1}^m(|v_j|_\infty + |z_j|_\infty),$$

$$|u_i|_\infty = \sup_{t\in\mathbb{R}}|u_i(t)|, \quad |y_i|_\infty = \sup_{t\in\mathbb{R}}|y_i(t)|, \quad i = 1,\dots,n,$$

$$|v_j|_\infty = \sup_{t\in\mathbb{R}} |v_j(t)|, \quad |z_j|_\infty = \sup_{t\in\mathbb{R}} |z_j(t)|, \quad j = 1,\ldots,m.$$

Then $\mathbb{X}$ is a Banach space when it is endowed with the norm $\|\cdot\|_{\mathbb{X}}$.

Let $x = (u_1,\ldots,u_n,y_1,\ldots,y_n,v_1,\ldots,v_m,z_1,\ldots,z_m)^T \in \mathbb{X}$. Define a linear operator $L\colon \operatorname{Dom} L = \{x : x\in\mathbb{X}, x'\in\mathbb{X}\} \subset \mathbb{X} \to \mathbb{X}$ by $Lx = x'$.

From (H_1), it is easy to see that for every $x\in\mathbb{X}$, the functions F_i, G_i, U_j, H_j defined in (2.2) are anti-periodic functions. So, we can define a continuous operator $N\colon \mathbb{X}\to\mathbb{X}$ by

$$Nx = \big(F_1,\ldots,F_n,G_1,\ldots,G_n,U_1,\ldots,U_m,H_1,\ldots,H_m\big)^T, \quad x\in\mathbb{X}.$$

It is easy to see that

$$\ker L = \{\mathbf{0}\} \quad \text{and} \quad \operatorname{Im} L = \left\{x\in\mathbb{X} : \int_0^{2\omega} x(s)\,ds = \mathbf{0}\right\} = \mathbb{X}.$$

So, $L\colon \operatorname{Dom} L \to \mathbb{X}$ is a one-to-one mapping. Denote by $L^{-1}\colon \mathbb{X}\to\operatorname{Dom} L$ the inverse of L, one has

$$\big(L^{-1}x\big)(t) = \int_0^t x(s)\,ds - \frac{1}{2}\int_0^\omega x(s)\,ds, \quad t\in\mathbb{R}.$$

Taking $K := L^{-1}N$, by applying the Arzela–Ascoli theorem, we know that K is compact.

Theorem 3.1. *Let* (H_1) *and* (H_2) *hold. Assume further that the following condition holds*:

(H_3) $1-2\omega(1+A) > 0$, *where*

$$A = \max_{\substack{1\le i\le n\\ 1\le j\le m}} \left\{|a_i-\alpha_i+1| + \sum_{j=1}^m (r_{ji}+p_{ji}+q_{ji})L_g, \right. \\ \left. |b_j-\beta_j+1| + \sum_{i=1}^n (c_{ij}+d_{ij}+e_{ij})L_f\right\}.$$

Then system (1.1) *has at least one* ω*-anti-periodic solution.*

Proof. Consider the operator equation $Lx = \lambda Nx, \lambda\in(0,1)$. Let $x\in\operatorname{Dom} L\subset\mathbb{X}$ be a solution of $Lx=\lambda Nx$ for a certain $\lambda\in(0,1)$. Then, we have

$$\begin{cases} u_i'(t) = \lambda F_i(u_i(t),y_i(t)), \\ y_i'(t) = \lambda G_i(t,u_i(t),y_i(t),v(t)), \\ v_j'(t) = \lambda U_j(v_j(t),z_j(t)), \\ z_j'(t) = \lambda H_j(t,u,v_j(t),z_j(t)), \end{cases} \quad i=1,\ldots,n,\ j=1,2,\ldots,m. \tag{3.1}$$

Since $x\in\mathbb{X}$ is ω-anti-periodic, there exist constants $\xi_i,\zeta_i,\eta_j,\mu_j\in[0,2\omega]$ such that

$$u_i(\xi_i) = y_i(\zeta_i) = v_j(\eta_j) = z_j(\mu_j) = 0, \quad i=1,\ldots,n,\ j=1,\ldots,m.$$

Next, integrating both sides of (3.1) from 0 to 2ω, we obtain

$$\int_0^{2\omega} |u_i'(s)|\,ds = \lambda \int_0^{2\omega} |-u_i(s)+y_i(s)|\,ds \le 2\omega(|u_i|_\infty + |y_i|_\infty),$$

$$\begin{aligned}
\int_0^{2\omega} |y_i'(s)|\,ds &\le \int_0^{2\omega} |a_i-\alpha_i+1||u_i(t)|\,dt + \int_0^{2\omega} |\alpha_i-1||y_i(t)|\,dt \\
&\quad + \int_0^{2\omega} \sum_{j=1}^m |c_{ij}(t)||f_j(v_j(t))-f_j(0)|\,dt + \int_0^{2\omega} \sum_{j=1}^m |c_{ij}(t)||f_j(0)|\,dt \\
&\quad + \int_0^{2\omega} \left| \bigwedge_{j=1}^m d_{ij}(t) f_j(v_j(t-\tau_{ji}(t))) - \bigwedge_{j=1}^m d_{ij}(t) f_j(0) \right| dt \\
&\quad + \int_0^{2\omega} \left| \bigvee_{j=1}^m e_{ij}(t) f_j(v_j(t-\tau_{ji}(t))) - \bigvee_{j=1}^m e_{ij}(t) f_j(0) \right| dt \\
&\quad + \int_0^{2\omega} \left| \bigwedge_{j=1}^m d_{ij}(t) f_j(0) \right| dt + \int_0^{2\omega} \left| \bigvee_{j=1}^m e_{ij}(t) f_j(0) \right| dt + \int_0^{2\omega} |I_i(t)|\,dt \\
&\le \int_0^{2\omega} |a_i-\alpha_i+1||u_i(t)|\,dt + \int_0^{2\omega} |\alpha_i-1||y_i(t)|\,dt \\
&\quad + \int_0^{2\omega} \sum_{j=1}^m c_{ij} L_f |v_j(t))|\,dt + \int_0^{2\omega} \sum_{j=1}^m d_{ij} |f_j(v_j(t-\tau_{ji}(t))) - f_j(0)|\,dt \\
&\quad + \int_0^{2\omega} \sum_{j=1}^m e_{ij} |f_j(v_j(t-\tau_{ji}(t))) - f_j(0)|\,dt + \int_0^{2\omega} \sum_{j=1}^m c_{ij} \overline{L}_f\,dt \\
&\quad + \int_0^{2\omega} \sum_{j=1}^m d_{ij} \overline{L}_f\,dt + \int_0^{2\omega} \sum_{j=1}^m e_{ij} \overline{L}_f\,dt + \int_0^{2\omega} |I_i(t)|\,dt \\
&\le 2\omega \Bigg\{ |a_i-\alpha_i+1||u_i|_\infty + |\alpha_i-1||y_i|_\infty \\
&\quad + \sum_{j=1}^m (c_{ij}+d_{ij}+e_{ij}) L_f |v_j|_\infty + \sum_{j=1}^m (c_{ij}+d_{ij}+e_{ij}) \overline{L}_f + I_i \Bigg\},
\end{aligned}$$

$$\int_0^{2\omega} |v_j'(s)|\,ds \le \int_0^{2\omega} \big(|v_j(s)| + |z_j(s)|\big)\,ds \le 2\omega(|v_j|_\infty + |z_j|_\infty),$$

$$\begin{aligned}
\int_0^{2\omega} |z_j'(s)|\,ds &\le \int_0^{2\omega} |b_j-\beta_j+1||v_j(s)|\,ds + \int_0^{2\omega} |\beta_j-1||z_j(s)|\,ds \\
&\quad + \int_0^{2\omega} \sum_{i=1}^n |r_{ji}(s)||g_i(u_i(s))-g_i(0)|\,ds + \int_0^{2\omega} \sum_{i=1}^n |r_{ji}(s)||g_i(0)|\,ds \\
&\quad + \int_0^{2\omega} \left| \bigwedge_{i=1}^n p_{ji}(s) g_i(u_i(s-\sigma_{ij}(s))) - \bigwedge_{i=1}^n p_{ji}(s) g_i(0) \right| ds
\end{aligned}$$

$$
\begin{aligned}
&\quad + \int_0^{2\omega} \bigg| \bigvee_{i=1}^{n} q_{ji}(s) g_i(u_i(s-\sigma_{ij}(s))) - \bigvee_{i=1}^{n} q_{ji}(s) g_i(0) \bigg| \, ds \\
&\quad + \int_0^{2\omega} \bigg| \bigwedge_{i=1}^{n} p_{ji}(s) g_i(0) \bigg| \, ds + \int_0^{2\omega} \bigg| \bigvee_{i=1}^{n} q_{ji}(s) g_i(0) \bigg| \, ds + \int_0^{2\omega} |J_j(s)| \, ds \\
&\le \int_0^{2\omega} |b_j - \beta_j + 1| |v_j(s)| \, ds + \int_0^{2\omega} |\beta_j - 1| |z_j(s)| \, ds \\
&\le 2\omega \bigg\{ |b_j - \beta_j + 1| |v_j|_\infty + |\beta_j - 1| |z_j|_\infty \\
&\quad + \sum_{i=1}^{n} (r_{ji} + p_{ji} + q_{ji}) L_g |u_i|_\infty + \sum_{i=1}^{n} (r_{ji} + p_{ji} + q_{ji}) \overline{L}_g + J_j \bigg\}.
\end{aligned}
$$

Since

$$
|u_i(t)| = \bigg| u_i(\xi_i) + \int_{\xi_i}^{t} u_i'(s) ds \bigg| \le \int_0^{2\omega} |u_i'(s)| \, ds,
$$

we have

$$
|u_i|_\infty \le \int_0^{2\omega} |u_i'(s)| \, ds \le 2\omega (|u_i|_\infty + |y_i|_\infty).
$$

Similarly, we have

$$
\begin{aligned}
|y_i|_\infty &\le 2\omega \bigg\{ |a_i - \alpha_i + 1| |u_i|_\infty + |\alpha_i - 1| |y_i|_\infty \\
&\qquad + \sum_{j=1}^{m} (c_{ij} + d_{ij} + e_{ij}) L_f |v_j|_\infty + \sum_{j=1}^{m} (c_{ij} + d_{ij} + e_{ij}) \overline{L}_f + I_i \bigg\}, \\
|v_j|_\infty &\le \int_0^{2\omega} |v_j'(s)| \, ds \le 2\omega (|v_j|_\infty + |z_j|_\infty), \\
|z_j|_\infty &\le 2\omega \bigg\{ |b_j - \beta_j + 1| |v_j|_\infty + |\beta_j - 1| |z_j|_\infty \\
&\qquad + \sum_{i=1}^{n} (r_{ji} + p_{ji} + q_{ji}) L_g |u_i|_\infty + \sum_{i=1}^{n} (r_{ji} + p_{ji} + q_{ji}) \overline{L}_g + J_j \bigg\}.
\end{aligned}
$$

Thus

$$
\begin{aligned}
\|x\|_{\mathbb{X}} &\le \sum_{i=1}^{n} 2\omega (|u_i|_\infty + |y_i|_\infty) + \sum_{j=1}^{m} 2\omega (|v_j|_\infty + |z_j|_\infty) \\
&\quad + \sum_{i=1}^{n} 2\omega \bigg\{ |a_i - \alpha_i + 1| |u_i|_\infty + |\alpha_i - 1| |y_i|_\infty \\
&\quad + \sum_{j=1}^{m} (c_{ij} + d_{ij} + e_{ij}) L_f |v_j|_\infty + \sum_{j=1}^{m} (c_{ij} + d_{ij} + e_{ij}) \overline{L}_f + I_i \bigg\} \\
&\quad + \sum_{j=1}^{m} 2\omega \bigg\{ |b_j - \beta_j + 1| |v_j|_\infty + |\beta_j - 1| |z_j|_\infty
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i=1}^{n}(r_{ji}+p_{ji}+q_{ji})L_g|u_i|_\infty+\sum_{i=1}^{n}(r_{ji}+p_{ji}+q_{ji})\overline{L}_g+J_j\Bigg\}\\
&=2\omega\|x\|_{\mathbb{X}}\\
&\quad+\sum_{i=1}^{n}2\omega\Bigg\{\Bigg[|a_i-\alpha_i+1|+\sum_{j=1}^{m}(r_{ji}+p_{ji}+q_{ji})L_g\Bigg]|u_i|_\infty+|\alpha_i-1||y_i|_\infty\Bigg\}\\
&\quad+\sum_{j=1}^{m}2\omega\Bigg\{\Bigg[|b_j-\beta_j+1|+\sum_{i=1}^{n}(c_{ij}+d_{ij}+e_{ij})L_f\Bigg]|v_j|_\infty+|\beta_j-1||z_j|_\infty\Bigg\}\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}2\omega[(c_{ij}+d_{ij}+e_{ij})\overline{L}_f+(r_{ji}+p_{ji}+q_{ji})\overline{L}_g]\\
&\quad+\sum_{i=1}^{n}2\omega I_i+\sum_{j=1}^{m}2\omega J_j\\
&\leq 2\omega\|x\|_{\mathbb{X}}+2\omega A\Bigg(\sum_{i=1}^{n}(|u_i|_\infty+|y_i|_\infty)+\sum_{j=1}^{m}(|v_j|_\infty+|z_j|_\infty)\Bigg)\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{m}2\omega[(c_{ij}+d_{ij}+e_{ij})\overline{L}_f+(r_{ji}+p_{ji}+q_{ji})\overline{L}_g]\\
&\quad+\sum_{i=1}^{n}2\omega I_i+\sum_{j=1}^{m}2\omega J_j\\
&=2\omega(1+A)\|x\|_{\mathbb{X}}+\sum_{i=1}^{n}\sum_{j=1}^{m}2\omega[(c_{ij}+d_{ij}+e_{ij})\overline{L}_f+(r_{ji}+p_{ji}+q_{ji})\overline{L}_g]\\
&\quad+\sum_{i=1}^{n}2\omega I_i+\sum_{j=1}^{m}2\omega J_j,
\end{aligned}
$$

or

$$
\begin{aligned}
\|x\|_{\mathbb{X}}\leq\frac{1}{1-2\omega(1+A)}\Bigg(&\sum_{i=1}^{n}\sum_{j=1}^{m}2\omega\big[(c_{ij}+d_{ij}+e_{ij})\overline{L}_f+(r_{ji}+p_{ji}+q_{ji})\overline{L}_g\big]\\
&+\sum_{i=1}^{n}2\omega I_i+\sum_{j=1}^{m}2\omega J_j\Bigg)\triangleq D.
\end{aligned}
$$

Take $\Omega=\{x\in\mathbb{X}:\|x\|_{\mathbb{X}}<D+1\}$, then $\Omega\subset\mathbb{X}$ with $0\in\Omega$ such that equation $Lx=\lambda Nx$ has no solutions in $\partial\Omega\cap\operatorname{Dom}L$ for any $\lambda\in(0,1)$. Hence, all the conditions of Lemma 2.4 are verified. Therefore, by Lemma 2.4 we conclude that $Lx=Nx$ has at least one ω-anti-solution in $\mathbb{X}$. That is, (2.2) has at least one ω-anti-periodic solution. In view of Remark 2.2, we see that (1.1) has at least one ω-anti-periodic solution. The proof is complete.

Theorem 3.2. *Let* (H_1) *and* (H_2') *hold. Assume that the following condition holds*:

(H$'_3$) $A' = \min\limits_{\substack{1\le i\le n\\ 1\le j\le m}} \left\{ 1 - \dfrac{\omega^2}{\pi^2}\,|a_i - \alpha_i + 1|,\ 1 - \dfrac{\omega^2}{\pi^2}\,|b_j - \beta_j + 1| \right\} > 0.$

Then system (1.1) *has at least one ω-anti-periodic solution.*

Proof. If $x \in \operatorname{Dom} L \subset \mathbb{X}$ is a solution of $Lx = \lambda N x$ for a certain $\lambda \in (0,1)$. Then we have (3.1) holds. Multiplying by $u'_i(t)$ on both sides of the first equation of (3.1) and then integrating it from 0 to 2ω, we have

$$\begin{aligned}\int_0^{2\omega} |u'_i(t)|^2\,dt &= \lambda \int_0^{2\omega} \big(-u_i(t) + y_i(t)\big)u'_i(t)\,dt = \lambda \int_0^{2\omega} y_i(t)u'_i(t)\,dt \\ &\le \bigg(\int_0^{2\omega} |u'_i(t)|^2\,dt\bigg)^{1/2} \bigg(\int_0^{2\omega} |y_i(t)|^2\,dt\bigg)^{1/2},\end{aligned}$$

for $i = 1,\ldots,n$, that is,

$$\bigg(\int_0^{2\omega} |u'_i(t)|^2\,dt\bigg)^{1/2} \le \bigg(\int_0^{2\omega} |y_i(t)|^2 dt\bigg)^{1/2}, \quad i = 1,\ldots,n. \tag{3.2}$$

Multiplying by $y'_i(t)$ on both sides of the second equation of (3.1) and then integrating it from 0 to 2ω, we have

$$\begin{aligned}\int_0^{2\omega} |y'_i(t)|^2\,dt = \lambda \int_0^{2\omega} \bigg[&-(a_i - \alpha_i + 1)u_i(t)y'_i(t) - (\alpha_i - 1)y_i(t)y'_i(t) \\ &+ \sum_{j=1}^m c_{ij}(t)f_j(v_j(t))y'_i(t) + \bigwedge_{j=1}^m d_{ij}(t)f_j(v_j(t-\tau_{ji}(t)))y'_i(t) \\ &+ \bigvee_{j=1}^m e_{ij}(t)f_j(v_j(t-\tau_{ji}(t)))y'_i(t) + I_i(t)y'_i(t)\bigg]\,dt \\ \le{}& \int_0^{2\omega} |a_i - \alpha_i + 1||u_i(t)y'_i(t)|\,dt \\ &+ \int_0^{2\omega} \sum_{j=1}^m |c_{ij}(t)||f_j(v_j(t))||y'_i(t)|\,dt \\ &+ \int_0^{2\omega} \sum_{j=1}^m |d_{ij}(t)||f_j(v_j(t-\tau_{ji}(t)))||y'_i(t)|\,dt \\ &+ \int_0^{2\omega} \sum_{j=1}^m |e_{ij}(t)||f_j(v_j(t-\tau_{ji}(t)))||y'_i(t)|\,dt \\ &+ \int_0^{2\omega} |I_i(t)||y'_i(t)|\,dt \\ \le{}& |a_i - \alpha_i + 1|\bigg(\int_0^{2\omega} |u_i(t)|^2\,dt\bigg)^{1/2} \bigg(\int_0^{2\omega} |y'_i(t)|^2\,dt\bigg)^{1/2} \\ &+ \sum_{j=1}^m \sqrt{2\omega}(c_{ij} + d_{ij} + e_{ij})N_f \bigg(\int_0^{2\omega} |y'_i(t)|^2\,dt\bigg)^{1/2}\end{aligned}$$

$$+\sqrt{2\omega}I_i\Big(\int_0^{2\omega}|y_i'(t)|^2\,dt\Big)^{1/2},$$

for $i=1,\dots,n$, that is,

$$\begin{aligned}\Big(\int_0^{2\omega}|y_i'(t)|^2\,dt\Big)^{1/2} \le{}& |a_i-\alpha_i+1|\Big(\int_0^{2\omega}|u_i(t)|^2\,dt\Big)^{1}/2 \\ &+\sum_{j=1}^{m}\sqrt{2\omega}(c_{ij}+d_{ij}+e_{ij})N_f+\sqrt{2\omega}I_i,\end{aligned}\tag{3.3}$$

for $i=1,\dots,n$. Since $u_i\in C^1$ and $u_i(0)=u_i(2\omega)=-u_i(\omega)$, by Lemma 2.5, for $i=1,\dots,n$, we have

$$\Big(\int_0^{2\omega}|u_i(t)|^2\,dt\Big)^{1/2}\le\frac{\omega}{\pi}\Big(\int_0^{2\omega}|u_i'(t)|^2\,dt\Big)^{1/2},$$

Similarly, we can get

$$\Big(\int_0^{2\omega}|y_i(t)|^2\,dt\Big)^{1/2}\le\frac{\omega}{\pi}\Big(\int_0^{2\omega}|y_i'(t)|^2\,dt\Big)^{1/2},\quad i=1,\dots,n.\tag{3.4}$$

From (3.2)–(3.4) it follows that, for $i=1,\dots,n$,

$$\Big(\int_0^{2\omega}|y_i'(t)|^2\,dt\Big)^{1/2}\le\frac{\sum_{j=1}^{m}\sqrt{2\omega}(c_{ij}+d_{ij}+e_{ij})N_f+\sqrt{2\omega}I_i}{1-\frac{\omega^2}{\pi^2}|a_i-\alpha_i+1|},$$

$$\Big(\int_0^{2\omega}|u_i'(t)|^2\,dt\Big)^{1/2}\le\frac{\sum_{j=1}^{m}\sqrt{2\omega}(c_{ij}+d_{ij}+e_{ij})N_f+\sqrt{2\omega}I_i}{\frac{\pi}{\omega}-\frac{\omega}{\pi}|a_i-\alpha_i+1|}.$$

Since $x=(u,y,v,z)^T\in\mathbb{X}$ is an ω-anti-periodic function, there exist positive constants ξ_i, ζ_i, η_j, μ_j such that

$$u_i(\xi_i)=y_i(\zeta_i)=v_j(\eta_j)=z_j(\mu_j)=0,\quad i=1,\dots,n,\ j=1,\dots,m.$$

By Newton Leibniz formula, for $i=1,\dots,n$, we have

$$\begin{aligned}|u_i|_\infty&\le\int_0^{2\omega}|u_i'(t)|\,dt\le\sqrt{2\omega}\Big(\int_0^{2\omega}|u_i'(t)|^2\,dt\Big)^{1/2}\\ &\le\frac{\sum_{j=1}^{m}2\omega(c_{ij}+d_{ij}+e_{ij})N_f+2\omega I_i}{\frac{\pi}{\omega}-\frac{\omega}{\pi}|a_i-\alpha_i+1|}\triangleq M_{1i},\end{aligned}$$

$$|y_i|_\infty \le \frac{\displaystyle\sum_{j=1}^{m} 2\omega(c_{ij}+d_{ij}+e_{ij})N_f + 2\omega I_i}{1-\dfrac{\omega^2}{\pi^2}|a_i-\alpha_i+1|} \triangleq M_{2i}.$$

Repeating the above reasoning process, one can get, for $j=1,2,\dots,m$,

$$|v_j|_\infty \le \frac{\displaystyle\sum_{i=1}^{n} 2\omega(r_{ji}+p_{ji}+q_{ji})N_g + 2\omega J_j}{\dfrac{\pi}{\omega}-\dfrac{\omega}{\pi}|b_j-\beta_j+1|} \triangleq M_{1j},$$

$$|z_j|_\infty \le \frac{\displaystyle\sum_{i=1}^{n} 2\omega(r_{ji}+p_{ji}+q_{ji})N_g + 2\omega J_j}{1-\dfrac{\omega^2}{\pi^2}|b_j-\beta_j+1|} \triangleq M_{2j}.$$

Hence,

$$\|x\|_{\mathbb{X}} = \sum_{i=1}^{n}(|u_i|_\infty+|y_i|_\infty) + \sum_{j=1}^{m}(|v_j|_\infty+|z_j|_\infty) \le (2(m+n))M \triangleq D',$$

where $M = \max\limits_{\substack{1\le i\le n\\ 1\le j\le m}} \{M_{1i}, M_{2i}, M_{1j}, M_{2j}\}$. Take $\Omega = \{x\in\mathbb{X} : \|x\|_{\mathbb{X}} < D'+1\}$. Thus, by Lemma 2.4 we see that (1.1) has at least one ω-anti-periodic solution.□

Theorem 3.3. *Assume that* (H_1) *and* (H_2) *hold. Furthermore, suppose that*

(H_4) *For* $i=1,\dots,n$, $j=1,\dots,m$, $\tau_{ji},\sigma_{ij}\in C^1(\mathbb{R},\mathbb{R}^+)$ *and*

$$\nu = \max_{\substack{1\le i\le n\\ 1\le j\le m}} \Big\{ \sup_{t\in\mathbb{R}} \dot\tau_{ji}(t),\ \sup_{t\in\mathbb{R}} \dot\sigma_{ij}(t) \Big\} < 1.$$

(H_5) *There exists a positive constant* $\varepsilon>0$ *satisfying*

$$\begin{aligned}\Gamma = \max_{\substack{1\le i\le n\\ 1\le j\le m}} \Bigg\{ & \varepsilon+2-\alpha_i,\\ & \varepsilon-1+|a_i-\alpha_i+1| + \sum_{j=1}^{m}\bigg(r_{ji}L_g + \frac{(p_{ji}+q_{ji})L_g e^{\varepsilon\tau}}{1-\nu}\bigg),\\ & \varepsilon+2-\beta_j,\\ & \varepsilon-1+|b_j-\beta_j+1| + \sum_{i=1}^{n}\bigg(c_{ij}L_f + \frac{(d_{ij}+e_{ij})L_f e^{\varepsilon\tau}}{1-\nu}\bigg)\Bigg\} < 0.\end{aligned}$$

Then every solution of system (1.1) *is globally exponentially stable.*

Proof. Let x and x^* be two arbitrary solutions of system (2.2) with the initial values $\Phi(t)$ and $\Phi^*(t)$, respectively. By (2.2), for $i=1,\dots,n$, $j=1,\dots,m$, we have

$$
\begin{aligned}
&D^+(e^{\varepsilon t}|u_i(t) - u_i^*(t)|)\\
&\qquad \leq \varepsilon e^{\varepsilon t}|u_i(t) - u_i^*(t)| + e^{\varepsilon t}\big[-|u_i(t) - u_i^*(t)| + |y_i(t) - y_i^*(t)|\big]\\
&\qquad \leq (\varepsilon - 1)e^{\varepsilon t}|u_i(t) - u_i^*(t)| + e^{\varepsilon t}|y_i(t) - y_i^*(t)|,\\
&D^+(e^{\varepsilon t}|y_i(t) - y_i^*(t)|)\\
&\qquad \leq \varepsilon e^{\varepsilon t}|y_i(t) - y_i^*(t)| + e^{\varepsilon t}\bigg\{|a_i - \alpha_i + 1||u_i(t) - u_i^*(t)|\\
&\qquad\quad - (\alpha_i - 1)|y_i(t) - y_i^*(t)| + \sum_{j=1}^{m} c_{ij}|f_j(v_j(t)) - f_j(v_j^*(t))|\\
&\qquad\quad + \bigg|\bigwedge_{j=1}^{m} d_{ij}(t)f_j(v_j(t - \tau_{ji}(t))) - \bigwedge_{j=1}^{m} d_{ij}(t)f_j^*(v_j(t - \tau_{ji}(t)))\bigg|\\
&\qquad\quad + \bigg|\bigvee_{j=1}^{m} e_{ij}(t)f_j(v_j(t - \tau_{ji}(t))) - \bigvee_{j=1}^{m} e_{ij}(t)f_j^*(v_j(t - \tau_{ji}(t)))\bigg|\bigg\}\\
&\qquad \leq \varepsilon e^{\varepsilon t}|y_i(t) - y_i^*(t)| + e^{\varepsilon t}\bigg\{|a_i - \alpha_i + 1||u_i(t) - u_i^*(t)|\\
&\qquad\quad - (\alpha_i - 1)|y_i(t) - y_i^*(t)| + \sum_{j=1}^{m} c_{ij}L_f|v_j(t) - v_j^*(t)|\\
&\qquad\quad + \sum_{j=1}^{m}(d_{ij} + e_{ij})L_f|v_j(t - \tau_{ji}(t)) - v_j^*(t - \tau_{ji}(t))|\bigg\}\\
&\qquad = e^{\varepsilon t}\bigg\{(\varepsilon + 1 - \alpha_i)|y_i(t) - y_i^*(t)| + |a_i - \alpha_i + 1||u_i(t) - u_i^*(t)|\\
&\qquad\quad + \sum_{j=1}^{m} c_{ij}L_f|v_j(t) - v_j^*(t)|\\
&\qquad\quad + \sum_{j=1}^{m}(d_{ij} + e_{ij})L_f|v_j(t - \tau_{ji}(t))v_j^*(t - \tau_{ji}(t))|\bigg\},\\
&D^+(e^{\varepsilon t}|v_j(t) - v_j^*(t)|)\\
&\qquad \leq \varepsilon e^{\varepsilon t}|v_j(t) - v_j^*(t)| + e^{\varepsilon t}[-|v_j(t) - v_j^*(t)| + |z_j(t) - z_j^*(t)|]\\
&\qquad \leq (\varepsilon - 1)e^{\varepsilon t}|v_j(t) - v_j^*(t)| + e^{\varepsilon t}|z_j(t) - z_j^*(t)|,\\
&D^+(e^{\varepsilon t}|z_j(t) - z_j^*(t)|) \leq \varepsilon e^{\varepsilon t}|z_j(t) - z_j^*(t)| + e^{\varepsilon t}\bigg\{|b_j - \beta_j + 1||v_j(t) - v_j^*(t)|\\
&\qquad\quad - (\beta_j - 1)|z_j(t) - z_j^*(t)| + \sum_{i=1}^{n} r_{ji}L_g|u_i(t) - u_i^*(t)|\\
&\qquad\quad + \sum_{i=1}^{n}(p_{ji} + q_{ji})L_g|u_i(t - \sigma_{ij}(t)) - u_i^*(t - \sigma_{ij}(t))|\bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&= e^{\varepsilon t}\bigg\{(\varepsilon + 1 - \beta_j)|z_j(t) - z_j^*(t)| + |b_j - \beta_j + 1||v_j(t) - v_j^*(t)| \\
&\quad + \sum_{i=1}^{n} r_{ji}L_g|u_i(t) - u_i^*(t)| \\
&\quad + \sum_{i=1}^{n}(p_{ji} + q_{ji})L_g|u_i(t - \sigma_{ij}(t)) - u_i^*(t - \sigma_{ij}(t))|\bigg\}.
\end{aligned}
$$

Define a Lyapunov function as follow.

$$
\begin{aligned}
V(t) = \sum_{i=1}^{n}&\bigg\{e^{\varepsilon t}(|u_i(t) - u_i^*(t)| + |y_i(t) - y_i^*(t)|) \\
&+ \sum_{j=1}^{m}\frac{(d_{ij} + e_{ij})L_f e^{\varepsilon\tau}}{1 - \nu}\int_{t-\tau_{ji}(t)}^{t} e^{\varepsilon s}|v_j(s) - v_j^*(s)|\,ds\bigg\} \\
&+ \sum_{j=1}^{m}\bigg\{e^{\varepsilon t}(|v_j(t) - v_j^*(t)| + |z_j(t) - z_j^*(t)|) \\
&+ \sum_{i=1}^{n}\frac{(p_{ji} + q_{ji})L_g e^{\varepsilon\tau}}{1 - \nu}\int_{t-\sigma_{ij}(t)}^{t} e^{\varepsilon s}|u_i(s) - u_i^*(s)|\,ds\bigg\}.
\end{aligned}
$$

Now we calculate the derivative of $V(t)$ along the solutions of system (2.2), we get

$$
\begin{aligned}
D^+V(t) \le \sum_{i=1}^{n}&\bigg\{D^+(e^{\varepsilon t}|u_i(t) - u_i^*(t)|) + D^+(e^{\varepsilon t}|y_i(t) - y_i^*(t)|) \\
&+ \sum_{j=1}^{m}\frac{(d_{ij} + e_{ij})L_f e^{\varepsilon\tau}}{1 - \nu}e^{\varepsilon t}|v_j(t) - v_j^*(t)| \\
&- \sum_{j=1}^{m}\frac{(d_{ij} + e_{ij})L_f e^{\varepsilon\tau}(1 - \dot{\tau}_{ji}(t))}{1 - \nu} \\
&\times e^{\varepsilon(t-\tau_{ji}(t))}|v_j(t - \tau_{ji}(t)) - v_j^*(t - \tau_{ji}(t))|\bigg\} \\
&+ \sum_{j=1}^{m}\bigg\{D^+(e^{\varepsilon t}|v_j(t) - v_j^*(t)|) + D^+(e^{\varepsilon t}|z_j(t) - z_j^*(t)|) \\
&+ \sum_{i=1}^{n}\frac{(p_{ji} + q_{ji})L_g e^{\varepsilon\sigma}}{1 - \nu}e^{\varepsilon t}|u_i(t) - u_i^*(t)| \\
&- \sum_{i=1}^{n}\frac{(p_{ji} + q_{ji})L_g e^{\varepsilon\sigma}(1 - \dot{\sigma}_{ij}(t))}{1 - \nu} \\
&\times e^{\varepsilon(t-\sigma_{ij}(t))}|u_i(t - \sigma_{ij}(t)) - u_i^*(t - \sigma_{ij}(t))| \\
\le \sum_{i=1}^{n}&e^{\varepsilon t}\bigg\{(\varepsilon - 1 + |a_i - \alpha_i + 1|)|u_i(t) - u_i^*(t)| \\
&+ (\varepsilon + 2 - \alpha_i)|y_i(t) - y_i^*(t)|
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{m}\left(c_{ij}L_f+\frac{(d_{ij}+e_{ij})L_f e^{\varepsilon\tau}}{1-\nu}\right)|v_j(t)-v_j^*(t)|\Bigg\}\\
&+\sum_{j=1}^{m}e^{\varepsilon t}\Bigg\{(\varepsilon-1+|b_j-\beta_j+1|)|v_j(t)-v_j^*(t)|\\
&+(\varepsilon+2-\beta_j)|z_j(t)-z_j^*(t)|\\
&+\sum_{i=1}^{n}\left(r_{ji}L_g+\frac{(p_{ji}+q_{ji})L_g e^{\varepsilon\tau}}{1-\nu}\right)|u_i(t)-u_i^*(t)|\Bigg\}\\
={}&\sum_{i=1}^{n}e^{\varepsilon t}\Bigg\{\Bigg[\varepsilon-1+|a_i-\alpha_i+1|\\
&+\sum_{j=1}^{m}\left(r_{ji}L_g+\frac{(p_{ji}+q_{ji})L_g e^{\varepsilon\tau}}{1-\nu}\right)\Bigg]|u_i(t)-u_i^*(t)|\\
&+(\varepsilon+2-\alpha_i)|y_i(t)-y_i^*(t)|\Bigg\}\\
&+\sum_{j=1}^{m}e^{\varepsilon t}\Bigg\{\Bigg[\varepsilon-1+|b_j-\beta_j+1|\\
&+\sum_{i=1}^{n}\left(c_{ij}L_f+\frac{(d_{ij}+e_{ij})L_f e^{\varepsilon\tau}}{1-\nu}\right)\Bigg]|v_j(t)-v_j^*(t)|\\
&+(\varepsilon+2-\beta_j)|z_j(t)-z_j^*(t)|\Bigg\}\\
\le{}&\Gamma e^{\varepsilon t}\Bigg\{\sum_{i=1}^{n}(|u_i(t)-u_i^*(t)|+|y_i(t)-y_i^*(t)|)\\
&+\sum_{j=1}^{m}(|v_j(t)-v_j^*(t)|+|z_j(t)-z_j^*(t)|)\Bigg\}\le 0.
\end{aligned}
$$

That is, for $t\ge 0$, $V(t)\le V(0)$. By the expression of $V(t)$, we obtain

$$
\begin{aligned}
V(t)\ge{}&\sum_{i=1}^{n}e^{\varepsilon t}(|u_i(t)-u_i^*(t)|+|y_i(t)-y_i^*(t)|)\\
&+\sum_{j=1}^{m}e^{\varepsilon t}(|v_j(t)-v_j^*(t)|+|z_j(t)-z_j^*(t)|)\\
\ge{}&e^{\varepsilon t}\Bigg\{\sum_{i=1}^{n}(|u_i(t)-u_i^*(t)|+|y_i(t)-y_i^*(t)|)\\
&+\sum_{j=1}^{m}(|v_j(t)-v_j^*(t)|+|z_j(t)-z_j^*(t)|)\Bigg\}=e^{\varepsilon t}\|x(t)-x^*(t))\|
\end{aligned}
$$

and

$$
\begin{aligned}
V(0) = {} & \sum_{i=1}^{n} \bigg\{ |u_i(0) - u_i^*(0)| + |y_i(0) - y_i^*(0)| \\
& + \sum_{j=1}^{m} \frac{(d_{ij} + e_{ij}) L_f e^{\varepsilon\tau}}{1-\nu} \int_{-\tau_{ji}(0)}^{0} e^{\varepsilon s} |v_j(s) - v_j^*(s)|\, ds \bigg\} \\
& + \sum_{j=1}^{m} \bigg\{ |v_j(0) - v_j^*(0)| + |z_j(0) - z_j^*(0)| \\
& + \sum_{i=1}^{n} \frac{(p_{ji} + q_{ji}) L_g e^{\varepsilon\tau}}{1-\nu} \int_{-\sigma_{ij}(0)}^{0} e^{\varepsilon s} |u_i(s) - u_i^*(s)|\, ds \bigg\} \\
\le {} & \sum_{i=1}^{n} \bigg\{ \bigg(1 + \sum_{j=1}^{m} \frac{(p_{ji} + q_{ji}) L_g \tau e^{\varepsilon\tau}}{1-\nu} \bigg) \sup_{t \in [-\tau, 0]} |u_i(s) - u_i^*(s)| \\
& + \sup_{t \in [-\tau, 0]} |y_i(s) - y_i^*(s)| \bigg\} \\
& + \sum_{j=1}^{m} \bigg\{ \bigg(1 + \sum_{i=1}^{n} \frac{(d_{ij} + e_{ij}) L_f \tau e^{\varepsilon\tau}}{1-\nu} \bigg) \sup_{t \in [-\tau, 0]} |v_j(s) - v_j^*(s)| \\
& + \sup_{t \in [-\tau, 0]} |z_j(s) - z_j^*(s)| \bigg\} \\
\le {} & M \sum_{i=1}^{n} \bigg\{ \sup_{s \in [-\tau, 0]} (|\varphi_{ui}(s) - \varphi_{ui}^*(s)| + \sup_{s \in [-\tau, 0]} |\phi_{yi}(s) - \phi_{yi}^*(s)| \bigg\} \\
& + \sum_{j=1}^{m} \bigg\{ \sup_{s \in [-\tau, 0]} |\varphi_{vj}(s) - \varphi_{vj}^*(s)| + \sup_{s \in [-\tau, 0]} |\phi_{zj}(s) - \phi_{zj}^*(s)| \bigg\} \\
= {} & M \|\Phi - \Phi^*\|,
\end{aligned}
$$

where

$$
M = \max_{\substack{1 \le i \le n \\ 1 \le j \le m}} \bigg\{ 1 + \sum_{i=1}^{n} \frac{(d_{ij} + e_{ij}) L_f \tau e^{\varepsilon\tau}}{1-\nu},\ 1 + \sum_{j=1}^{m} \frac{(p_{ji} + q_{ji}) L_g \tau e^{\varepsilon\tau}}{1-\nu} \bigg\} > 0,
$$

thus, we have

$$
\|x(t) - x^*(t)\| \le M \|\Phi - \Phi^*\| e^{-\varepsilon t}, \quad t > 0.
$$

Therefore, every solution of system (2.2) is globally exponentially stable. By Remark 2.2, we know that every solution of system (1.1) is also globally exponentially stable. □

The following two corollaries are direct results of Theorems 3.1–3.3.

Corollary 3.4. *If* (H_1)–(H_5) hold. *Then system* (1.1) *has a unique* ω*-anti-periodic solution and it is globally exponentially stable.*

Corollary 3.5. *If* (H_1), (H_2), (H_2'), (H_3'), (H_4) *and* (H_5) *hold. Then system* (1.1) *has a unique* ω*-anti-periodic solution and it is globally exponentially stable.*

4. Numerical examples

In this section, two examples are given to illustrate the effectiveness of our results in this paper.

Example 4.1. In system (1.1), let $m = n = 2$ and take

$$
\begin{aligned}
&f_1(v) = f_2(v) = \frac{1}{12}v, \qquad && g_1(u) = g_2(u) = \frac{1}{12}u,\\
&\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 3, && a_1 = a_2 = b_1 = b_2 = 2,\\
&\tau_{11}(t) = \tau_{12}(t) = \tau_{21}(t) = \tau_{21}(t) = \frac{1}{30}\sin^2(15t), &&\\
&\sigma_{11}(t) = \sigma_{12}(t) = \sigma_{21}(t) = \sigma_{22}(t) = \frac{1}{30}\cos^2(15t), &&\\
&d_{ij}(t) = \frac{9}{16}\sin(30t), && e_{ij}(t) = \frac{7}{8}\sin^2(15t),\\
&p_{ji}(t) = \cos(30t), && q_{ji}(t) = \frac{5}{6}\cos^2(15t),
\end{aligned}
$$

$$
\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} \sin(15t) \\ \cos(15t) \end{bmatrix}, \qquad \begin{bmatrix} J_1(t) \\ J_2(t) \end{bmatrix} = \begin{bmatrix} \sin(15t) \\ \cos(15t) \end{bmatrix},
$$

$$
\begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix} = \begin{bmatrix} \sin^2(15t) & \cos(30t) \\ \sin(30t) & \cos^2(15t) \end{bmatrix},
$$

$$
\begin{bmatrix} r_{11}(t) & r_{12}(t) \\ r_{21}(t) & r_{22}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{3}\cos^2(15t) & \frac{1}{3}\cos(30t) \\ \sin(30t) & 3\cos^2(30t) \end{bmatrix}.
$$

Then, it is easy to obtain that

$$
L_f = \frac{1}{12}, \qquad L_g = \frac{1}{12}, \qquad \omega = \frac{\pi}{15}, \qquad \tau = \frac{1}{30}, \qquad \nu = \frac{1}{2},
$$

$$
d_{11} = d_{12} = d_{21} = d_{22} = \frac{9}{16}, \qquad e_{11} = e_{12} = e_{21} = e_{22} = \frac{7}{8},
$$

$$
p_{11} = p_{12} = p_{21} = p_{22} = 1, \qquad q_{11} = q_{12} = q_{21} = q_{22} = \frac{5}{6},
$$

$$
\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \qquad \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 1 & 3 \end{bmatrix}.
$$

Thus,

$$A = \max_{1\le i,j\le 2}\bigg\{|a_i-\alpha_i+1| + \sum_{j=1}^{2}(r_{ji}+p_{ji}+q_{ji})L_g,$$
$$|b_j-\beta_j+1| + \sum_{i=1}^{2}(c_{ij}+d_{ij}+e_{ij})L_f\bigg\} = \frac{7}{12}.$$

$$1-2\omega(1+A)\approx 0.337>0.$$

Taking $\varepsilon = 0.01$, we can get

$$\begin{aligned}\Gamma = \max_{1\le i,j\le 2}\bigg\{&\varepsilon+2-\alpha_i,\\ &\varepsilon-1+|a_i-\alpha_i+1| + \sum_{j=1}^{2}\bigg(r_{ji}L_g + \frac{(p_{ji}+q_{ji})L_g e^{\varepsilon\tau}}{1-\nu}\bigg),\\ &\varepsilon+2-\beta_j,\\ &\varepsilon-1+|b_j-\beta_j+1| + \sum_{i=1}^{2}\bigg(c_{ij}L_f + \frac{(d_{ij}+e_{ij})L_f e^{\varepsilon\tau}}{1-\nu}\bigg)\bigg\}\\ \approx -0.1<0.\end{aligned}$$

By Corollary 3.4, system (1.1) has a unique $(\pi/15)$-anti-periodic solution and it is globally exponentially stable (see Figures 1–3).

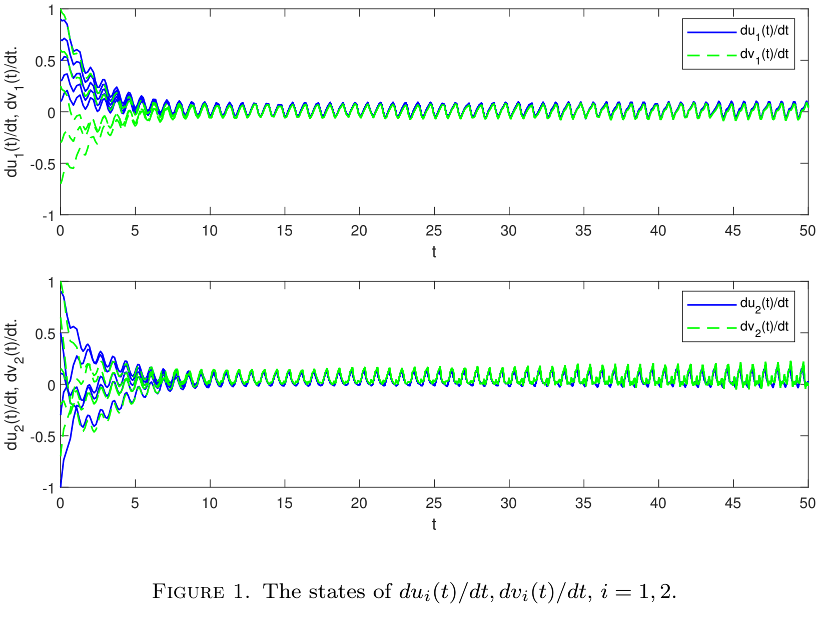

Figure 1. The states of $du_i(t)/dt, dv_i(t)/dt$, $i = 1, 2$.

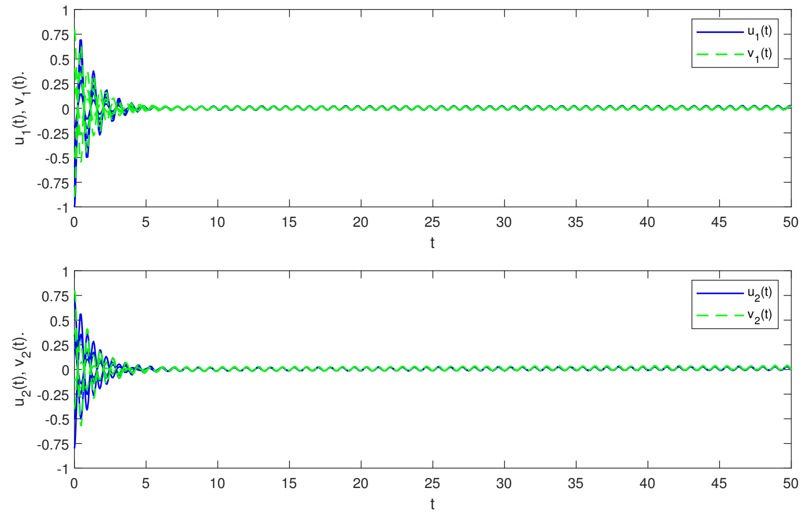

FIGURE 2. The states of $u_i(t), v_i(t)$, $i = 1, 2$.

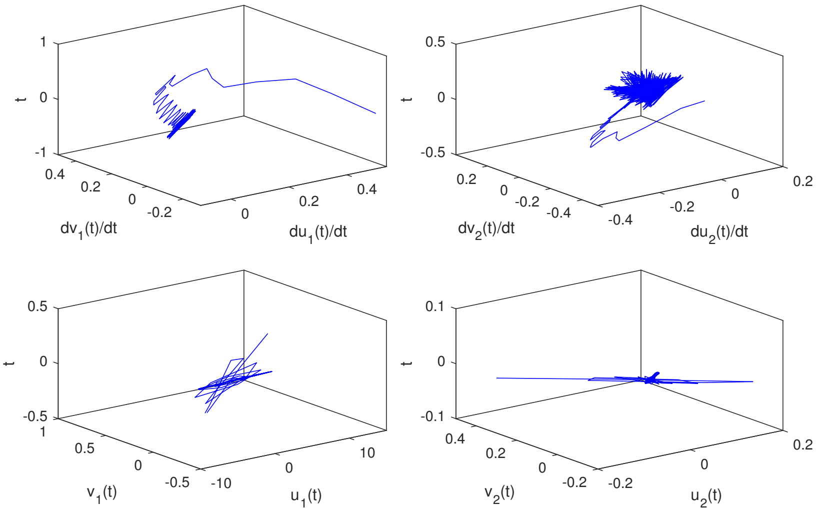

FIGURE 3. Curves of $u_i(t)$, $v_i(t)$, $du_i(t)/dt$, $dv_i(t)/dt$, $i = 1, 2$ in 3-dimensional space for stable case.

EXAMPLE 4.2. In system (1.1), let $m = n = 2$ and take

$$f_1(v) = f_2(v) = \frac{1}{4}\sin v, \qquad g_1(u) = g_2(u) = \frac{1}{4}\cos u,$$

$$\alpha_1 = \alpha_2 = 4, \qquad \beta_1 = \beta_2 = 3, \qquad a_1 = a_2 = 3, \qquad b_1 = b_2 = 2,$$

$$\tau_{ji}(t) = \frac{1}{4}\sin^2 t + \frac{1}{4}, \quad \sigma_{ij}(t) = \frac{1}{2}\cos^2 t, \qquad I_i(t) = \sin t, \quad J_j(t) = \cos t,$$

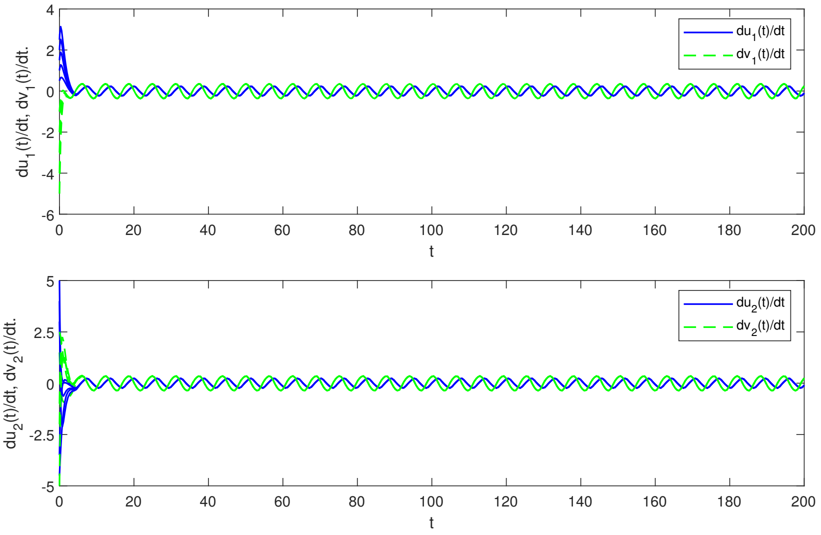

Figure 4. The states of $du_i(t)/dt$, $dv_i(t)/dt$, $i = 1, 2$.

$$c_{ij}(t) = \frac{1}{4}\sin(2t), \qquad d_{ij}(t) = \frac{1}{4}\cos(2t), \quad e_{ij}(t) = \frac{1}{8}\sin^2 t + \frac{1}{8},$$
$$r_{ji}(t) = \frac{1}{4}\sin t, \qquad p_{ji}(t) = \frac{1}{4}\cos t, \qquad q_{ji}(t) = \frac{1}{4}\sin(3t).$$

Then it is easy to see that f_j and $g_i(i, j = 1, 2)$ are bounded,

$$c_{ij} = d_{ij} = e_{ij} = r_{ji} = p_{ji} = q_{ji} = \frac{1}{4}, \quad \tau = \nu = \frac{1}{2}, \quad L_f = L_g = \frac{1}{4}, \quad \omega = \pi.$$

By computing,

$$A' = \min_{1\le i,j\le 2}\left\{1 - \frac{\omega^2}{\pi^2}|a_i - \alpha_i + 1|,\, 1 - \frac{\omega^2}{\pi^2}|b_j - \beta_j + 1|\right\} = 1 > 0.$$

Taking $\varepsilon = 0.1$, we can get

$$\begin{aligned}\Gamma = \max_{1\le i,j\le 2}\bigg\{&\varepsilon + 2 - \alpha_i,\, \varepsilon - 1 + |a_i - \alpha_i + 1| + \sum_{j=1}^{2}\left(r_{ji}L_g + \frac{(p_{ji} + q_{ji})L_g e^{\varepsilon\tau}}{1-\nu}\right),\\ &\varepsilon + 2 - \beta_j,\, \varepsilon - 1 + |b_j - \beta_j + 1| + \sum_{i=1}^{2}\left(c_{ij}L_f + \frac{(d_{ij} + e_{ij})L_f e^{\varepsilon\tau}}{1-\nu}\right)\bigg\}\\ \approx& -0.249 < 0.\end{aligned}$$

By Corollary 3.2, system (1.1) has a unique π-anti-periodic solution and it is globally exponentially stable (see Figures 4–6).

5. Conclusion

In this paper, a class of fuzzy BAM neural networks with inertial terms and time-varying delays was considered. By using a new continuation theorem of

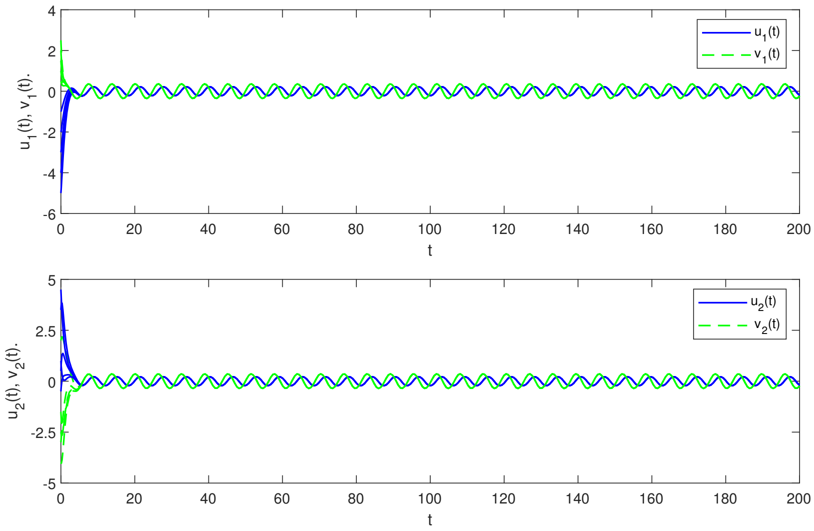

Figure 5. The states of $u_i(t), v_i(t)$, $i = 1, 2$.

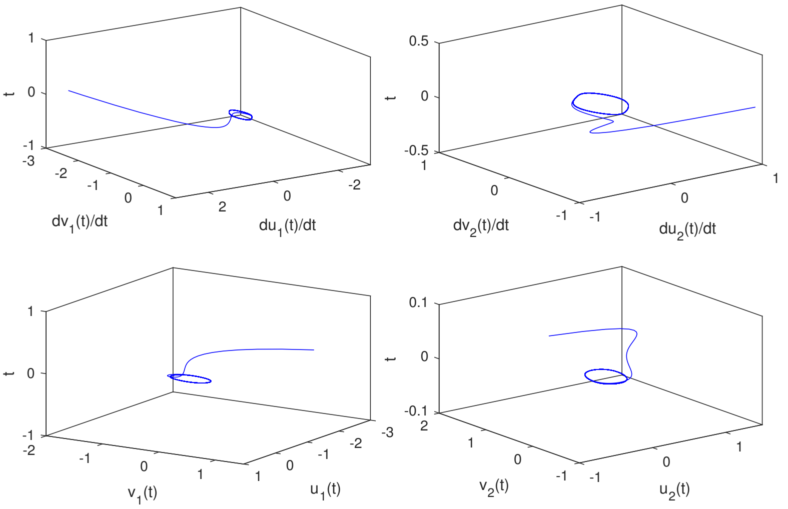

Figure 6. Curves of $u_i(t)$, $v_i(t)$, $du_i(t)/dt$, $dv_i(t)/dt$, $i = 1, 2$ in 3-dimensional space for stable case.

coincidence degree theory, the existence of anti-periodic solutions of the networks is obtained. By constructing an appropriate Lyapunov function, some sufficient conditions are derived to ensure the global exponential stability of anti-periodic solution of the networks. Our results of this paper are completely new and our methods used in the paper can be used to study other types' neural networks with or without inertial terms.

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Yongkun Li (the corresponding author)
`https://orcid.org/0000-0003-0182-6288`
Department of Mathematics
Yunnan University
Kunming, Yunnan 650091, P.R. CHINA
E-mail address: yklie@ynu.edu.cn

Jiali Qin
Department of Mathematics
Yunnan University
Kunming, Yunnan 650091, P.R. CHINA