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Nicolaus Copernicus University in Toruń

STEKLOV PROBLEMS FOR THE FOURTH ORDER HÉNON–HARDY EQUATION

Zongming Guo — Fangshu Wan

Abstract. Let Ω be a bounded smooth domain in $\mathbb{R}^N$ ($N \geq 5$) with $0 \in \Omega$. Non-trivial nonnegative weak solutions of Steklov problems of the fourth order Hénon–Hardy equation are obtained via variational methods and new embeddings related to new Caffarelli–Kohn–Nirenberg inequalities. Meanwhile, the further regularities of the weak solutions are studied, especially, the optimal regularity at the singular point $x = 0$ of the solutions is obtained.

1. Introduction

Let $\Omega \subset \mathbb{R}^N$ ($N \geq 5$) be a bounded smooth domain with $0 \in \Omega$. We are interested in positive solutions of Steklov problems for the fourth order Hénon–Hardy equation:

$$\begin{cases} \Delta^2 u = |x|^\alpha u^p & \text{in } \Omega, \\ \dfrac{\partial(\Delta u)}{\partial \nu} - au = 0, \quad \Delta u - b\dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\alpha, a, b \in \mathbb{R}$ with $\alpha > -4$, $p \geq 1$, ν is the unit outward normal vector of $\partial\Omega$.

Elliptic problems with parameters in the boundary conditions have been called Steklov or Steklov-type problems since they appeared in [18] firstly. For

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