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A THOM ISOTOPY THEOREM FOR NONPROPER SEMIALGEBRAIC MAPS

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Abstract. We prove a version of the Thom isotopy theorem for nonproper semialgebraic maps $f\colon X \to \mathbb{R}^m$, where $X \subset \mathbb{R}^n$ is a semialgebraic set and f is the restriction to X of a smooth semialgebraic map $F\colon \mathbb{R}^n \to \mathbb{R}^m$.

1. Introduction

Let $F\colon \mathbb{R}^n \to \mathbb{R}^m$ be a semialgebraic map. We say that F is locally trivial at $t_0 \in \mathbb{R}^m$ if there exist a neighbourhood U of t_0 and a diffeomorphism $h\colon F^{-1}(t_0)\times U \to F^{-1}(U)$ such that $F \circ h = \mathrm{pr}_2$, where $\mathrm{pr}_2\colon F^{-1}(t_0) \times U \to U$ denotes the second projection. The set of points $t_0 \in \mathbb{R}^m$ at which the local triviality of F fails is called the bifurcation set of F and is denoted by $B(F)$. It is well known that $B(F)$ is the union of the singular values set of F, denoted by $F(\operatorname{Sing} F)$, and the set of bifurcation values at infinity of F.

A full characterisation of $B(F)$ is a challenging open problem. In order to estimate $B(F)$, we can use conditions that control the asymptotic behaviour of the fibres of F. Rabier [11] considered the asymptotic critical values set $K_\infty(F)$

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