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A FUNCTION À LA HOPF–WHITNEY THAT DETECTS OR NOT STRONG SURJECTIVITY

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Abstract. Given a finite and connected two-dimensional complex K and a homomorphism $\beta \in \hom(\pi_1(K);\mathbb{Z}_2)$, we consider the function $\Phi_\beta\colon [K;\mathbb{R}\mathrm{P}^2]^*_\beta \to H^2(K;{}_\beta\mathbb{Z})$ defined by $[f]^* \mapsto f^*(\nu)$, where ν is a preferred generator of the twisted cohomology group $H^2(\mathbb{R}\mathrm{P}^2;{}_\varrho\mathbb{Z})$. We prove that Φ_β is a κ_β-to-one function, where κ_β is the order of the kernel of the multiplication by 2 on $H^2(K;{}_\beta\mathbb{Z})$, and we present necessary and sufficient conditions for both: Φ_β to be injective and Φ_β to be surjective. Furthermore, we prove that Φ_β detects strong surjectivity if and only if either $0 \notin \operatorname{im}(\Phi_\beta)$ or the set $[K;\mathbb{R}\mathrm{P}^2]^*_\beta$ contains κ_β classes having a representative given by a composite $K \to S^1 \hookrightarrow \mathbb{R}\mathrm{P}^2$.

1. Introduction

Let K be a finite and connected two-dimensional cw-complex (two-complex, for short) with fundamental group Π. For each homomorphism $\beta \in \hom(\Pi;\mathbb{Z}_2)$, we consider the set $\left[K;\mathbb{R}\mathrm{P}^2\right]^*_\beta$ of the based homotopy classes of maps inducing β on fundamental groups, and the twisted cohomology group $H^2(K;{}_\beta\mathbb{Z})$.

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