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A p-LAPLACIAN PROBLEM WITH SLIGHTLY SUBCRITICAL REGULARLY VARYING NONLINEARITY

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Abstract. We study a quasilinear elliptic problem involving the p-Laplacian operator and a slightly subcritical nonlinearity with a sign-changing weight. We assume that the slightly subcritical nonlinearity is a regularly varying function at zero and at infinity, which are not necessarily asymptotic to a power at infinity. We state sufficient conditions which guarantee a Palais–Smale condition. We also provide a bifurcation theorem for these nonlinearities, which allow us to state the existence of a bifurcated branch of positive solutions, containing a turning point, and multiplicity of solutions.

1. Introduction

We are interested in the existence of positive solutions to a Dirichlet problem for a class of quasilinear elliptic equations whose nonlinear term is slightly subcritical and regularly varying at infinity, see $(f1)_\infty$ and $(f3)_\infty$, and involves an indefinite weight. More precisely, we consider the quasilinear problem

$$-\Delta_p u = \lambda u^{p-1} + m(x)f(u) \quad \text{in } \Omega, \qquad u = 0, \quad \text{on } \partial\Omega, \tag{1.1}$$

where $1 < p < N$, $\Omega \subset \mathbb{R}^N$ is a bounded, connected open subset, with $C^{1,\alpha}$ boundary $\partial\Omega$, $\Delta_p(u) = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian operator, $\lambda \in \mathbb{R}$ is

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