Topological **M**ethods in **N**onlinear **A**nalysis
Volume 67, No. 2, 2026, 573–588
DOI: 10.12775/TMNA.2025.046

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A NONLOCAL LOGISTIC EQUATION WITH NONLINEAR ADVECTION TERM

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Abstract. In this paper, we study a nonlocal logistic equation with nonlinear advection term

$$(\mathrm{P}_p)\quad \begin{cases} -\Delta u + \overrightarrow{\alpha}(x)\cdot\nabla(u^p) = \left(\lambda - \displaystyle\int_\Omega K(x,y)u^\gamma(y)\,dy\right)u & \text{in } \Omega,\\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with smooth boundary, $\overline{\alpha}(x) = (\alpha_1(x), \dots, \alpha_N(x))$ is a flow satisfying suitable condition, $\gamma > 0$, $p \geq 1$, $\lambda \in \mathbb{R}$ and $K\colon \Omega \times \Omega \to \mathbb{R}$ is a nonnegative function with $K \in L^\infty(\Omega \times \Omega)$ and verifying other conditions that will be detailed below. It is very important to note that, this equation is not the classic logistic equation due to the inclusion of the term $\overline{\alpha}(x)\cdot\nabla(u^p)$, moreover, the inclusion of the integral nonlocal term on the right-hand side makes the problem closer to a real world situation.

2020 *Mathematics Subject Classification.* 35B32, 35B50, 35J60.

Key words and phrases. Bifurcations in context of PDE's; maximum principles in context of PDE's; nonlinear elliptic equations.

M.A.S. Souto was partially supported by CNPq/Brazil 309.692/2020-2 and grant 3031/2021 FAPESQ-PB.

R.N. de Lima was partially supported by CNPq/Brazil 306.411/2022-9 and grant 3031/2021 and 3177/2021 FAPESQ-PB.