Topological **M**ethods in **N**onlinear **A**nalysis
Volume 67, No. 2, 2026, 547–572
DOI: 10.12775/TMNA.2025.059

© 2026 Juliusz Schauder Centre for Nonlinear Studies
Nicolaus Copernicus University in Toruń

ON CHOQUARD–KIRCHHOFF TYPE CRITICAL MULTIPHASE PROBLEM

Anupma Arora — Gaurav Dwivedi

Abstract. This article focuses on the study of the following Choquard–Kirchhoff type critical multiphase problem:

$$\begin{aligned} &-M(\varphi_{\mathcal{T}}(|\nabla u|)) \\ &\qquad \times \operatorname{div}\big(|\nabla u|^{p(x)-2}\nabla u + a_1(x)|\nabla u|^{q(x)-2}\nabla u + a_2(x)|\nabla u|^{r(x)-2}\nabla u\big) \\ &\quad = \lambda g(x)|u|^{\gamma(x)-2}u + \theta B(x,u) \\ &\qquad + \kappa\left(\int_\Omega \frac{F(y,u(y))}{|x-y|^{d(x,y)}}\,dy\right) f(x,u) && \text{in } \Omega, \\ &u = 0 && \text{on } \partial\Omega. \end{aligned}$$

Here, the nonlinearity $B(x,u)$ exhibits critical growth. To handle this critical growth, we present the concentration compactness principle in the space $W_0^{1,\mathcal{T}}(\Omega)$. To address the Choquard term, we prove the Hardy–Littlewood–Sobolev-type inequality in the framework of the generalized Sobolev space $W_0^{1,\mathcal{T}}(\Omega)$. These tools, combined with variational methods, are used to establish the existence and multiplicity of weak solutions.

2020 *Mathematics Subject Classification.* Primary: 35J20, 35B33; Secondary: 35D30, 35J60, 35J15.

Key words and phrases. Multiphase operator with variable exponents; critical growth; Kirchhoff problem; Choquard nonlinearity; Musielak–Orlicz–Sobolev spaces.

The first author thanks the University Grants Commission, India, for providing financial support (Ref. No. 191620115768).