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COMPACTNESS IN LORENTZ SEQUENCE SPACES

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Abstract. In this paper we are going to discuss compactness in Lorentz sequence spaces. Firstly, it will be shown how to define such a space, check whether a sequence belongs to it and calculate its norm. Equipped with this knowledge, we will proceed to propose usable compactness criteria for Lorentz sequence spaces, employing the concept of seminorms.

1. Introduction

Lorentz spaces were defined in year 1950 by George Gunter Lorentz in the paper [6] as functional spaces consisting of measurable functions on a bounded interval of $\mathbb{R}$. This definition can be naturally generalised to all possible spaces with measure, as in [3]. This source expands significantly on the known uses of Lorentz spaces and possibilities of further research.

As a special case, the space with measure can be chosen as $\mathbb{N}$ with the counting measure, thus defining a $l_{p,q}$ space. It is discussed in the article [2]. A somewhat more general version, with a weight sequence w, is given by [1]. Thus defined *Lorentz sequence spaces* will be of main interest in our further considerations.

They can also be defined in a different but equivalent manner, through permutations, as in [5]. It is mentioned there that the concept of *nonincreasing rearrangement* can be used to conduct the definition more smoothly. In the later sections here, it will be described precisely by the means of injective maps,

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