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ON EXISTENCE OF COMMON FIXED POINTS FOR POINTWISE EVENTUALLY NONEXPANSIVE MAPPINGS

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Abstract. Muoi and Wong proved that a finite and commuting family of weakly continuous pointwise eventually nonexpansive mappings from E into itself has a common fixed point in E whenever E is a nonempty weakly compact convex subset of a Banach space B. Without the assumption of weak continuity of pointwise eventually nonexpansive mappings, we prove the existence of common fixed points for a commuting family of pointwise eventually nonexpansive mappings in uniformly convex in every direction Banach spaces, k-uniformly rotund Banach spaces, and nearly uniformly convex Banach spaces. This improves the result of Muoi and Wong and extends the fixed point result of Domínguez Benavides and Lorenzo Ramírez to a commuting family of pointwise eventually nonexpansive mappings. Moreover, we prove that a left reversible semigroup of pointwise eventually nonexpansive mappings from E into itself has a common fixed point in E whenever E is a nonempty weakly compact convex subset of a uniformly convex in every direction Banach space B. This extends the fixed point result of S. Rajesh to a left reversible semigroup of pointwise eventually nonexpansive mappings.

1. Introduction

Kirk and Xu [11] introduced the notion of pointwise eventually nonexpansive mappings as follows:

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Key words and phrases. Pointwise eventually nonexpansive mappings; left reversible semigroup; common fixed point; UCED Banach spaces; URE_k Banach spaces; property $(\mathcal{P})$.

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