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THE REIDEMEISTER AND THE NIELSEN NUMBERS: GROWTH RATE, ASYMPTOTIC BEHAVIOR, DYNAMICAL ZETA FUNCTIONS AND THE GAUSS CONGRUENCES

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Abstract. In the present paper, taking a dynamical point on view, we study the growth rate and asymptotic behavior of the sequences of the Reidemeister numbers and the sequences of the Reidemeister and the Nielsen coincidence numbers. We also prove the Gauss congruences for the sequence $\{R(\varphi^n, \psi^n)\}$ of the Reidemeister coincidence numbers of the tame pair (φ, ψ) of endomorphisms of a torsion-free nilpotent group G of finite Prüfer rank. Furthermore, we prove the rationality of the Nielsen coincidence zeta function, the Gauss congruences for the sequence $\{N(f^n, g^n)\}$ of the Nielsen coincidence numbers and show that the growth rate exists for the sequence $\{N(f^n, g^n)\}$ of tame pair of maps (f, g) of a compact nilmanifold to itself.

1. Introduction

In topological fixed point theory, the Reidemeister and the Nielsen numbers arise as homotopy invariants associated to a continuous self-map of a connected compact polyhedron. Passing to the fundamental group of the polyhedron, the Reidemeister numbers admit an algebraic treatment in terms of twisted conjugacy classes; compare [7]. We assume X to be a connected, compact polyhedron and $f\colon X \to X$ to be a continuous map. Let $p\colon \widetilde{X} \to X$ be the universal cover

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