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A NEW CONSTRUCTION OF COUNTEREXAMPLES TO THE BOUNDED ORBIT CONJECTURE

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Abstract. The bounded orbit conjecture says that every homeomorphism on the plane with each of its orbits being bounded must have a fixed point. Brouwer's translation theorem asserts that the conjecture is true for orientation preserving homeomorphisms, but Boyles' counterexample shows that it is false for the orientation reversing case. In this paper, we give a more comprehensible construction of counterexamples to the conjecture. Roughly speaking, we construct an orientation reversing homeomorphism f on the square $J^2 = [-1,1]^2$ with $\omega(x,f) = \{(-1,1),(1,1)\}$ and $\alpha(x,f) = \{(-1,-1),(1,-1)\}$ for each $x \in (-1,1)^2$. Then, by a semi-conjugacy defined by pushing an appropriate part of ∂J^2 into $(-1,1)^2$, f induces a homeomorphism on the plane, which is a counterexample.

1. Introduction

In 1912, Brouwer [2] obtained his translation theorem, which has been a fundamental tool in studying surface homeomorphisms nowadays (see e.g. [7]).

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