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A TWO-PHASE FREE BOUNDARY PROBLEM INVOLVING EXPONENTIAL OPERATOR

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Abstract. In this paper we are interested in the study of a two-phase problem equipped with the Φ-Laplacian operator

$$\Delta_\Phi u := \operatorname{div}\left(\varphi(|\nabla u|)\,\frac{\nabla u}{|\nabla u|}\right),$$

where $\Phi(s) = e^{s^2} - 1$ and $\varphi = \Phi'$. We obtain the existence, boundedness, and Log–Lipschitz regularity of the minimizers of the energy functional associated to the two-phase problem. Furthermore, we also prove that the free boundaries of these minimizers have locally finite perimeter and Hausdorff dimension at most $(N-1)$.

1. Introduction

In the last decades, there has been a great increase in interest in the study of two-phase partial differential equations arising from multiphase flow in porous media, phase transmission in materials science and the dynamics of biological systems. Important examples of two-phase problems include multiphase flow in porous media, where understanding the behavior of different fluids (e.g. water,

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